

What Can I Do with a Math Degree?

Quantitative Analyst, D. E. Shaw Group

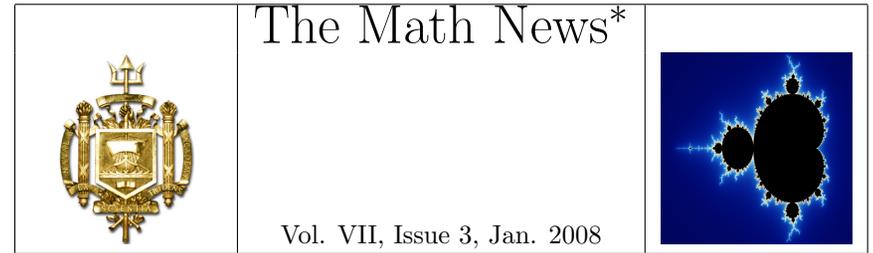
- **Company Overview:** D. E. Shaw (<http://www.deshaw.com>) is a global investment and technology development firm with 1,300 employees, \$35 billion in aggregate investment capital, and offices in N. America, Europe, and Asia. Since its organization in 1988, the firm has earned an international reputation for financial innovation, technological leadership, and an extraordinarily distinguished staff. Its activities range from the deployment of investment strategies based on either mathematical models or human expertise to the acquisition of existing companies and the financing or development of new ones.
- **Job Description:** Quants apply mathematical techniques and write software to develop and analyze statistical models for our computerized financial trading strategies. Responsibilities range from examining trading data in an effort to increase profitability, decrease risk, and reduce transaction costs to conceiving new trading ideas and devising the simulations needed to test them. Successful quant candidates have been the top students in their math, physics, engineering, and CS programs; a considerable number have also competed successfully in the Math Olympiads as well as the Putnam Competition.

Problem of the Month

- **Jan. Problem:** Find all *prime triplets*—numbers n , $n + 2$, and $n + 4$ that are all prime. Prove that your list is complete. Mids who submit a correct solution to the editor via email will be entered in a random drawing for Math Department swag.
- **Nov. Solution:** Since $f(x)f(f(x)) = 1$ for all $x \in \mathbb{R}$ and $f(1000) = 999$, $1 = f(1000)f(f(1000)) = 999f(999)$. Thus, $f(999) = 1/999$. Since f is continuous, the Intermediate Value Theorem guarantees that there is a $c \in (999, 1000)$ such that $f(c) = 500$. Then $1 = f(c)f(f(c)) = 500f(500)$. Therefore, $f(500) = 1/500$. A correct solution was submitted by MIDN 4/C Housseem Marzougui. Come claim your prize!

Contact the Editor

Asst. Prof. Vrej Zarikian (zarikian@usna.edu)
Department of Mathematics



Number Theory

by Assoc. Prof. Will Traves (traves@usna.edu)

Number theory is once again a hot topic. Facts about prime numbers were used to develop public-key cryptography, the system that enables secure communication over the internet. We understand the prime numbers better than at any time in the past, but there remain many curious open questions concerning their behavior. This semester midshipmen in the number theory course SM463 will study how calculus, geometry, and algebra all play a role in the study of numbers.

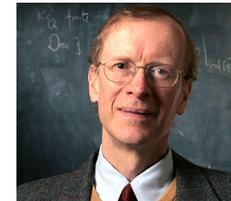
The ancient Greeks proved that there are infinitely many prime numbers[†] and the celebrated *Prime Number Theorem* predicts that the n th prime number is close to $n/\ln(n)$.

The Greek philosopher Pythagoras studied positive integers a , b , c satisfying $a^2 + b^2 = c^2$. Some examples are $(a, b, c) = (3, 4, 5)$ or $(5, 12, 13)$. But in fact there are infinitely many *Pythagorean triples*. In 1637 the French judge Pierre de Fermat claimed that if the exponent in Pythagoras's equation is changed to an integer higher than 2, then there are no positive integer solutions. This problem was something like the space program of mathematics—it is not so important on its own, but mathematicians developed whole new areas of mathematics to try to solve it. *Fermat's Last Theorem* was still open in 1995 when Princeton professor Andrew Wiles (pictured above) announced that he had found a proof covering hundreds of pages. His proof uses a subtle result on elliptic curves, special curves with cubic equations that are also used to encode secret communications.[‡]

*On the cover: The USNA seal (left) and the Mandelbrot set (right).

[†]If there is a largest prime n , then what are the prime divisors of $n! + 1$?

[‡]For the story of Wiles' proof, watch the NOVA documentary *The Proof* or visit <http://www.pbs.org/wgbh/nova/proof/>.



More recently, the mathematicians Ben Green and Terence Tao (pictured below) made a major advance in the theory of arithmetic progressions among the primes.



These are sequences of equally spaced prime numbers, such as 31, 37, 43 or 13, 43, 73, 103. Results in this area go back to the eighteenth century, but a major breakthrough came in 1939 when the Dutch mathematician van der Corput showed that there are an infinite number of three-term arithmetic progres-

sions involving only prime numbers. The *Green-Tao Theorem* states that there are infinitely many n -term arithmetic progressions among the primes, for arbitrarily large n .

Despite centuries of progress, deceptively simple questions remain open. *Goldbach's Conjecture* asks if every even number greater than 2 is the sum of two primes. And the *Twin Prime Conjecture* asks if there are infinitely many pairs of twin primes, primes like 11 and 13 that differ by just 2.

The mathematician G.H. Hardy claimed to like number theory because it was so pure that it could not find application. However, modern number theory is crucial to ensuring secure and secret communication.

Math in the News: Factoring $2^{1039} - 1$

According to *Discover* magazine's ranking (Jan. 2008), the 38th most important scientific achievement of 2007 was made by an international team of mathematicians led by Arjen Lenstra. They used the *special number field sieve* to find the prime factorization of $2^{1039} - 1$, a 300+ digit (Mersenne) number. The feat took hundreds of computers working in tandem 11 months to complete. Nonetheless, it sent a warning to computer security experts everywhere. Modern computer security is based on the RSA algorithm, which in turn is based on the fact that factoring a number which is the product of two 150+ digit (unknown) prime numbers is extremely difficult. Although $2^{1039} - 1$ is not such a number and factoring it took a Herculean computational effort, it would have been unthinkable 20 years ago. So while the current encryption standard is still secure, advances in both algorithms and computation mean that the clock is ticking.

Faculty Profile: Asst. Prof. Leah Jager

Assistant Professor Leah Jager was born and raised in Grand Rapids, Michigan. She attended Calvin College, where she majored in math and chemistry. Although she enjoyed her chemistry classes, she liked her math classes even better. As a result, she decided to pursue a graduate degree in a mathematical field — she chose statistics.



Professor Jager first became interested in statistics during a summer internship in a food lab within the R&D division at Amway Corporation. Her summer project was focused on making a beverage taste better by using a flavor additive. In order to determine whether or not the beverage actually tasted better, she had to run taste tests on other employees and analyze the data using some simple statistical models. She never succeeded in making the drink taste better, but she did become interested in statistics.

Professor Jager attended graduate school at the University of Washington in Seattle.

While in the Pacific Northwest, she often went hiking and camping to take advantage of the beautiful outdoors. She met her husband, Jeff Leek, playing intramural athletics at UW, where she was a member of the joint statistics/biostatistics teams that won an unprecedented 9 intramural championships.

Professor Jager's current research is in the area of goodness-of-fit testing, which involves determining the distribution of a set of data. For example, a professor who is thinking of curving grades on a test might be interested in knowing whether his students' test grades follow a bell-shaped, or normal, curve. Right now she is working on writing a computationally efficient software package to perform the new statistical tests she has developed. She is also working on applications of her new software, including applications in genomics, a project she is working on with her husband.

Mathematics and statistics is a theme in Professor Jager's family. Her father, oldest sister, and brother-in-law all have doctorates in mathematics, while her middle sister, husband, and herself all have doctorates in statistics. Her mother is an English teacher, who makes sure they all use proper grammar.

Apart from statistics, Professor Jager enjoys running, exploring the East Coast, and cooking.