



## Mathematical Models of Disease Spread

by LT Ben Heineike ([heineike@usna.edu](mailto:heineike@usna.edu))

When a disease breaks out in a population, policy makers at the Centers for Disease Control or the Armed Forces Health Surveillance Center must act quickly to slow its spread. They need fast answers to questions: How many people will be affected? What are the most effective control measures? Are the risks and costs of a mass vaccination program worthwhile? Should schools be closed and sporting events canceled? What about air travel?



In order to help make these decisions policy makers often turn to mathematical models. One basic but effective disease spread model is the *SIR model* devised by W.O. Kermack and A.G. McKendrick in 1927. The model places individuals in an affected population (size assumed constant) into one of three categories: *susceptible*, *infected*, and *removed*, with respective populations  $S(t)$ ,  $I(t)$ , and  $R(t)$ . Because new infections are created by interactions between susceptible and infected individuals, we model the

On the cover: The USNA seal (left) and the Mandelbrot set (right).  
 Photo credit: "Immune Defense" by Sebastian Kaulitzki  
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dynamics of the system with the following system of differential equations:

$$\frac{dS}{dt} = -rSI, \quad \frac{dI}{dt} = rSI - aI, \quad \frac{dR}{dt} = aI$$

where the parameter  $r$  is the *infection rate*, and  $a$  is the *removal rate*. We can also represent the model using the block diagram shown below:



In the SIR model, the number of infected individuals stops growing when  $S = \frac{a}{r}$ . For a given scenario with  $S(0) = S_0$ ,  $I(0) = I_0$ , and  $R(0) = 0$ , the disease will spread if  $S_0 > \frac{a}{r}$ . This has led epidemiologists to define the *basic reproductive ratio* for an epidemic event as  $R_0 = \frac{S_0 r}{a}$ , which can be interpreted as the number of secondary infections caused by an initial infected individual. If  $R_0 > 1$ , then the disease will spread, otherwise the total number of cases in a population will steadily decline. The form of the equation for  $R_0$  gives us insight into how to control the spread of a disease. In order to lower  $R_0$ , one can use one or more of three possible approaches:

1. Reduce  $S_0$ , the initial number of susceptible people in the population, through vaccination programs.
2. Lower the infection rate,  $r$ , by reducing social contact, providing face masks, or having people wash their hands more frequently.
3. Increase the removal rate,  $a$ , by having infected individuals rest and take medicine that helps them combat the infection more quickly.

Another interesting feature about this model is that the infection dies out before all the susceptible individuals in the population are taken ill. In other words,  $I_\infty = \lim_{t \rightarrow \infty} I(t) = 0$ . With this fact we can calculate  $S_\infty$ , the final number of susceptible individuals left in the population, by solving the equation  $I_0 + S_0 - S_\infty + \frac{a}{r} \ln\left(\frac{S_\infty}{S_0}\right) = 0$  for  $S_\infty$ . This computation can be performed using numerical methods similar to Newton's Method for finding roots. We can calculate the total number of individuals who were infected over the course of the disease using the fact that the population remains constant:  $R_\infty = I_0 + S_0 - S_\infty$ .

You can learn more about modeling disease spread in Mathematical Biology (SM450), a new math elective being offered by LT Ben Heineike ([heineike@usna.edu](mailto:heineike@usna.edu)) and Asst. Prof. Russell Jackson ([rkjackson@usna.edu](mailto:rkjackson@usna.edu)) during Fall 2010. You can also talk to MIDN 1/C Jonathan Rix, who is studying math models of disease spread for his math honors project.

## Prof. Konkowski wins Research Award



Prof. Deborah Konkowski of the USNA Math Department is the 2010 recipient of the *Class of 1951 Civilian Faculty Award for Excellence in Research*. This award is presented annually to the civilian faculty member who has exhibited the highest quality continued scholarly achievement through research. Prof. Konkowski joined the Math Department in 1987, and has research interests in the field of classical and quantum particle behavior in general relativistic spacetimes. With almost continuous funding from the National Science Foundation for the past 20 years, Prof. Konkowski has published 8 papers in the last five years and an additional 60 papers, conference proceedings, and abstracts over her career. In 2005, she was chosen by the American Physical Society as one of only fifty World Year of Physics speakers in the U.S. and gave talks on Einstein and his legacy. Congratulations, Debbie!

## Faculty Profile: CDR David Ruth



CDR David Ruth first arrived at the Naval Academy in the summer of 1987 from Corpus Christi, TX. He graduated with the Class of 1991 with a B.S. in Mathematics and followed on at the University of Texas at Austin to earn an M.A. in Mathematics in 1993. Since then he has served in both the Submarine and Oceanography communities, deploying across the globe and serving in various at-sea billets and international assignments. Each new Navy job has added to his appreciation of the application of mathematics to the sea service. Understanding mathematical principles and techniques has been a key component to success in:

- learning how to run a nuclear reactor and to operate a submarine at sea,
- comprehending the dynamics of weather, waves, and currents as an oceanographer,
- working through complicated operational and logistical details as a military planner on the Korean peninsula, and

- integrating quantitative analysis into the development of effective DoD strategies and policies, to name a few.

In 2006 CDR Ruth was selected to be a USNA Permanent Military Professor, and in 2009 he earned a Ph.D. in Operations Research from the Naval Postgraduate School. His dissertation research proposed new non-parametric approaches to the multidimensional change-point problem. He reported to USNA in November 2009, and currently teaches Calculus II.

CDR Ruth married his wife Leticia in 1993, and they have three children: Christian (8), Eliana (5), and Elizabeth (2). He especially enjoys family time, sports, music, and travel.

## Math Department Open House!

The Wall Street Journal says “Mathematician” is the best occupation in the United States\*, and we want to show you why. On

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you are invited to the Math Department Open House, which takes place  
on the 3rd deck of Chauvenet

All are welcome! Come learn about

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- SASMC (the Service Academy Student Mathematics Conference)
- Modeling, simulation, and internships in mathematics
- Completing language minors within mathematics
- Completing pre-medical/pre-dental programs within mathematics
- the USNA Math Team and problem-solving competitions

You can also win door prizes, match your chess prowess against Prof. Will Traves (who will take on all comers simultaneously), or just watch math movies on our medium-sized screen. We'll supply the ice cream and drinks!

\*<http://online.wsj.com/article/SB123119236117055127.html>