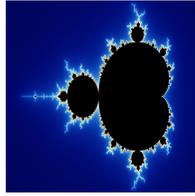




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Fibonacci Numbers

by Assoc. Prof. T. S. Michael (tsm@usna.edu)

The sequence of *Fibonacci numbers*

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

has captured the imaginations of professional and amateur mathematicians for over 800 years. The sequence starts with the two “seed values” $F_1 = 1$ and $F_2 = 1$. Each subsequent Fibonacci number is the sum of the previous two, as asserted by the *recurrence*

$$F_n = F_{n-1} + F_{n-2} \quad \text{for } n \geq 3.$$

The sequence of numbers is named after the Italian mathematician Fibonacci of Pisa (circa 1170–1250), the most prominent European mathematician of the Middle Ages.



On the cover: The USNA seal (left) and the Mandelbrot set (right).
Editors: T. S. Michael (tsm@usna.edu) and Vrej Zarikian (zarikian@usna.edu)

Fibonacci discussed these numbers in his book *Liber Abaci*, an influential work that introduced the Hindu-Arabic number system (with the digits 0–9) to Europe and demonstrated their superiority to the cumbersome Roman numerals.

Here are some of the many remarkable properties of Fibonacci numbers. First, observe that $5^2 - (3 \cdot 8) = 1$ and $8^2 - (5 \cdot 13) = -1$. In general,

$$F_n^2 - F_{n-1}F_{n+1} = (-1)^{n+1}.$$

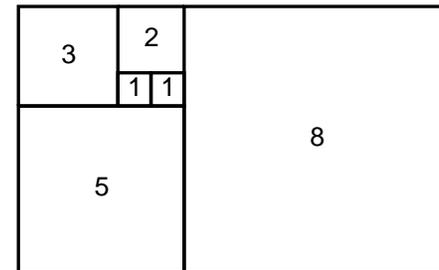
Second, observe that $1 + 2 + 5 = 8$ and $1 + 2 + 5 + 13 = 21$. In general, if we add every other Fibonacci number we get another Fibonacci number:

$$F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}.$$

Question: Can you discover what happens with the sum

$$F_2 + F_4 + F_6 + \cdots + F_{2n}?$$

Third, observe that $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 104 = 8 \cdot 13$. This numerical fact can be verified geometrically by assembling six squares to form an 8-by-13 rectangle, as in the figure.



In general, a rectangle can be formed using a similar spiral of Fibonacci squares to show that

$$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + \cdots + F_n^2 = F_n F_{n+1}.$$

We close by mentioning a surprising formula for the n -th Fibonacci number*:

$$F_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^n.$$

Midshipmen can learn more about this formula and other properties of Fibonacci numbers in SM291 (Fundamentals of Math) and SM342 (Discrete Mathematics).

* $\frac{1+\sqrt{5}}{2} \approx 1.6180$ is the *golden ratio*, also known as the *divine proportion*.

Faculty Profile: Asst. Prof. Russell Jackson

Asst. Prof. Russell Jackson moved around the southeast a lot as a youth (he attended 11 different schools between the start of Kindergarten and the completion of his Ph.D.), but he calls North Carolina his home. He stayed in North Carolina for his undergraduate degree, going to Duke University on a Math Contest Scholarship. During his freshman year, he signed up for a course in Symbolic Logic that the course catalog said was “open to first year students with the permission of the chair”, not realizing that it meant first year philosophy graduate students! (He stayed in the course anyway.)



During his time at Duke, he was a Cameron Crazy, camping out for basketball games, and even played center on his intra-mural basketball team. He finished his senior season with more made dunks than made free throws – not so hard, as he vividly remembers being a perfect zero out of thirteen (ouch) from the free throw line that year. He also sang with an a cappella group that toured churches, nursing homes, prisons and shelters from North Carolina to Hawaii. The group sang in Duke’s Wallace Wade Stadium and even in the Washington Cathedral (although, there, they were asked to stop, as they were meant to be taking a tour and not actually performing). Prof. Jackson finally left the

Southeast to go to graduate school in New England, receiving his Ph.D. in Applied Mathematics from Brown University. While at Brown, he spent a summer as an intern in the Mathematical Research Division at Bell Labs, where he helped develop a geometric rationale for the stability of light pulses in fiber-optic communication systems. As a post-doc in the Dynamical Systems group at Boston University, he helped to understand a model for epileptic brain activity, analyzing the propagation of stimuli through a network of neurons with non-local interactions.

Since joining the USNA math faculty in 2005, Prof. Jackson has taught a broad range of courses. Currently, he is working to develop a set of worksheets to help Calculus students take charge of their own learning. He is also putting together an activity-based e-book for the math majors’

course in partial differential equations (SM315). Meanwhile, he continues to pursue research into the properties of waves in various applications, and this semester is helping to advise a 1/C midshipman’s math honors thesis on modeling infectious disease spread in Bancroft Hall. Professor Jackson currently publishes the oft-deleted but seldom-duplicated *Brigade Math Problem of the Week* – look for it in your inbox on Monday mornings – and is also hoping to find an engineer to help him flesh out next year’s big Christmas gift sensation, the Möbius racetrack!

Prof. Jackson lives in Cape St. Claire with his wife, also a Blue Devil, and their two sons, Joseph and Zachariah (who were both born in Annapolis). He and his wife sing in their church’s motet ensemble, and they enjoy competing in triathlons. Prof. Jackson actually completed his first Ironman in Lake Placid, NY the week before he reported to the Academy.

Famous Math Majors

Thinking about majoring in mathematics? You’d be in fine company! The following famous individuals were all math majors once upon a time.

- **Corazon Aquino**, former president of the Philippines (1986-1992).
- **Harold Blackmun**, former U. S. Supreme Court Justice (1970-1994).
- **Sergey Brin**, co-founder of Google, Inc.
- **Lewis Carroll**, author of *Alice’s Adventures in Wonderland*.
- **Art Garfunkel**, musical partner of Paul Simon.
- **Matt Groenig**, creator of *The Simpsons*.
- **Reed Hastings**, founder of Netflix.
- **Teri Hatcher**, desperate housewife.
- **Danica McKellar**, former *The Wonder Years* actress (Winnie Cooper), author of *Math Doesn’t Suck: How to Survive Middle-School Math without Losing Your Mind or Breaking a Nail*.
- **Larry Niven**, author of the science fiction classic *Ringworld*.
- **William J. Perry**, former U. S. Secretary of Defense (1994-1997).
- **David Robinson**, “The Admiral”, USNA Class of 1987.
- **Virginia Wade**, former Wimbledon Women’s Singles Champion (1977). Last Briton to hold a Grand Slam singles title.