

**Activating Calculus to Command
the Seas: Reflecting on ten years of
active and inquiry-based learning at
the US Naval Academy**

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Activating Calculus to Command the Seas: Reflecting on ten years of active and inquiry-based learning at the US Naval Academy

Abstract: Over the past ten years, the authors of this paper have implemented a series of project-based and inquiry-based learning initiatives in the calculus sequence at our institution. These initiatives took place in a department with an existing strong culture supporting teaching innovation and improvement. In this paper we describe three initiatives in detail. We describe how in one, a syllabus redesign allowed for more instructor autonomy. In another, we were able to significantly improve student buy-in. We describe key support we received from our department and institutional leadership at each stage, as well as from working in community with each other. Finally, by looking at the long arc of change in our department, we hope to encourage others who are encountering roadblocks. By working in community, we have created an environment where active learning can flourish.

Keywords: active learning, inquiry-based learning, project-based learning, calculus, communities of practice

1 INTRODUCTION

Over the past ten years, the authors of this paper have implemented a series of project-based and inquiry-based learning initiatives in the cal-

culus sequence at our institution. Each one had successes, and each had failures from which we learned and improved. We now have a thriving community of instructors successfully implementing inquiry-based learning in calculus, alongside other instructors with more traditional teaching styles. At the same time, the department culture has gradually shifted, and active learning in many different forms has become increasingly common. In a recent short survey of our department, the majority of faculty members who responded indicated that they have increased their use of active learning and/or inquiry-based learning, for various reasons.

In this paper, we describe how we built on an existing community of practice around teaching to create an environment where active learning can flourish. We will describe how a syllabus redesign helped support a good balance between consistency in multi-section courses and instructor autonomy. We will also describe how we have tackled student resistance, and how the leaders of our department and institution were able to provide effective support. We will describe the materials we have used, and the smaller communities of practice that have grown up around three focused initiatives undertaken by the authors.

In the next section we will give background information on our department, describe the framework we are using to understand the changes we have experienced, and describe how we use the terms “active learning,” “inquiry-based learning,” and “project-based learning.” Section 3 will highlight the key lessons we have learned over the course of the past ten years, and takeaways we hope will be useful for other departments. Section 4 will give more details about our department’s ongoing culture of supporting teaching, and faculty professional development. Sections 5, 6, and 7 will provide further details about the three initiatives undertaken by the authors of this paper: The CAPABLE

project, a project-based version of Calculus II, and an inquiry-based version of Calculus I. Finally we add our parting conclusions in Section 8.

2 BACKGROUND

2.1 Context

Our mathematics department is situated in a four-year, engineering-focused undergraduate institution. The department has roughly 40 permanent faculty members and 20 instructors; most of the instructors are in 3 year fixed term positions and do not typically hold Ph.D.s. The permanent faculty all hold doctoral degrees. The research expectations and outputs of the permanent faculty are above that of most undergraduate-only four-year colleges and universities; faculty do research in pure and applied mathematics, statistics, and operations research. With a few exceptions, faculty teach 18-24 credit hours per academic year. Courses in the calculus sequence (including precalculus, multivariable calculus, and differential equations) make up about half of the teaching in the department, and this teaching is fairly evenly split between the permanent faculty and instructors. Class sizes are capped at 20-22 students per section. Failure rates in the calculus sequence are typically around 5% and withdrawal rates are near zero.

Teaching is central to the mission of our institution. However, as with most institutions of higher education, very few of our faculty have any formal instruction in pedagogy, and instructors arrive at our institution with a wide variety of experience in teaching. Our departmental culture values time and effort spent on improving teaching; many instructors make use of (and contribute to) local training and support resources, and otherwise find ways to engage in learning beyond the immediate context of their current courses [4]. Thus, our department has had a “community

of practice” around teaching for many years [38]. As with most effective communities of practice [40], the efforts we describe in Sections 5-7 were not initiated by the department leadership, although they did receive significant support. In our context, learning about teaching often comes from meeting and talking with more experienced practitioners in the department. The learning is informal and entirely voluntary. In some cases, it can also be transformative as instructors develop their own identities as teachers [30]. More details about our department’s supports for instructors are given in Section 4.

2.2 Active vs project-based vs inquiry-based

In this paper “active learning” refers to time spent during class on activities done by students. For example, active learning could be students working individually or in small groups on tasks, either at their desks, on the computer, at the board or any combination thereof. It could also be students presenting or speaking to the whole class.

“Inquiry-based learning” (IBL) is a branch of active learning [29]. There is not a hard defining line, but the “inquiry-based” branch of active learning has several key characteristics. First, the kinds of tasks that students are given to work on have a unique flavor. Inquiry tasks build on each other, and connect together in longer narrative arcs, oriented towards having students develop, or “discover,” the main ideas of the course. Students collaborate in this development, bouncing ideas off of each other and evaluating each others’ reasoning. Instructors pay close attention to students in order to understand their thinking, and use this understanding to inform instructional choices. This includes daily choices such as asking a particular student or group to share their reasoning with the class because that group’s work has a particular feature the instructor wants to highlight. It includes longer-term choices, like

deciding to spend more time on a topic students are struggling with or excited about, and giving briefer treatment to another topic to compensate. It also includes synthesizing what one has learned about student thinking in general, from experience and from the education literature, to inform larger choices, such as whether to teach series before or after integration, or how grades should be computed. In making all of these decisions, instructors have an opportunity and a responsibility to critically consider social dynamics in the classroom and to foster an equitable environment, in part by recognizing and valuing students' diverse contributions.

“Project-based learning” is a term that has been used in various ways in our department; ten years ago, it had a very narrow definition, referring to courses, or parts of courses, designed around multi-day, hands-on projects [34]. Many people who are familiar with multi-disciplinary projects at the K-12 level might consider our “projects” to be more in line with “problem-based learning” [15]. In Sections 5 and 6, we will describe how things evolved, so that students and faculty at our institution began referring to inquiry-based learning materials, which often stretched over multiple days, as “projects,” and now often use “project-based learning” to refer to the classes that were using those materials.

3 KEY TAKEAWAYS

3.1 Coordinating for a balance of autonomy and consistency

One feature of our department that presents both opportunities and challenges for those who want to change their teaching is the balance between coordination of multi-section calculus courses and instructor autonomy. Our largest calculus courses typically have at least 30 sections, which are taught by a dozen or more instructors. For over ten years, we have been using the same textbook ([33]) for all of our Cal-

culus I, II, and III courses. Each semester the course coordinator, an experienced (usually permanent) faculty member, writes a common syllabus and daily course calendar, including suggested homework problems for each day. The daily homework is coded into the online homework system affiliated with the textbook, and instructors can set it up to be graded automatically. Thus, instructors can save a lot of work by following the daily plan. There is also peer pressure within the department to “keep up with the syllabus.” In addition, the campus Academic Center offers a variety of very popular programs geared towards calculus students: tutoring, exam review sessions, peer-led group study sessions, and “extra help” classes that meet for one hour per week. All of these student resources follow the calendar set by the course coordinator. Thus, instructors have many incentives to follow the provided course calendar, and most faculty, especially new instructors, do.

However, autonomy is also highly valued in our department. Each instructor sets their own grading policy, and writes and schedules their own midterm tests. The coordinator writes a common final exam, but generally incorporates significant input from the other instructors. Half of the final exam is multiple choice and machine graded; the other half consists of free response questions. Each instructor grades their own students’ exams¹. This system does offer autonomy to the instructors, as long as their students are ready for the common final exam by the end of the semester. With this freedom, many instructors write their own activities and problem sets, change the order of material or add material, or implement different grading systems such as standards-based grading [32]. Our department leadership has also been adamant about not en-

¹Over the past ten years, there has been a push for more consistency. Individual instructors used to sometimes replace problems on the free response portion of the final; that is now frowned upon. Recently, some courses have held “grading parties” for the final exam, where each instructor grades one problem across many sections.

dorsing any particular teaching method, but encouraging instructors to do what they find works best for themselves and their students.

We have discovered two connected practices that are supportive of both consistency and autonomy. The first has been to have the course coordinator separate the “syllabus” from the “course calendar”. The syllabus gives course-wide policies and information, and includes a detailed list of learning outcomes (for example, “use the Ratio Test to find the open interval of convergence of a power series”). This list has become the reference for what instructors are responsible for covering, and what will appear on the final exam. (Previously, much of this information was not explicitly stated, but encoded in the coordinator’s choice of homework problems.) The course calendar then becomes a suggestion of one way to achieve these objectives, including daily topics, textbook sections, and homework problems. A second related practice provides a separate course calendar of suggested daily IBL activities and problem sets. In more recent semesters, our coordination of the calendars has improved so that students in both “traditional” and IBL courses are well served by Academic Center programs.

3.2 Student buy-in and support from departmental leadership

It is very common for students to resist a curriculum in which, probably for the first time in their educational careers, they are regularly expected to work on problems before being “taught how to do them” [21] [36] [42]. Some students naturally rise to this challenge, but many require at least some convincing. Lack of sufficient student buy-in can lead to poor effort on homework and less productive class meetings, reducing the overall effectiveness of the curriculum. It also erodes instructor morale.

We found lack of student buy-in to be a significant problem starting with our second initiative, a project-based version of Calculus II (see

Section 6). The support of our department chair was invaluable at this time; more details are in Section 6. Department leadership has continued to vocally support the pedagogical decisions of all faculty. They have also provided support by assigning teams of instructors to classes who plan to work together to adopt active curricula. In addition, departmental and institutional leaders have made it clear that they are aware that student evaluations can be adversely affected by the adoption of student-centered pedagogy, and that this will not be used against junior faculty applying for tenure or promotion.

Student buy-in has improved significantly since our first implementation of a project-based curriculum in Calculus II, and continues to improve. Over the past few years, through outside training [2], as well as the many supports listed in Section 4, we have developed several strategies instructors can use for achieving student buy-in. These start with effective messages at the beginning of the course; we now devote the first day of class to activities such as “Setting the Stage” [16], or an activity developed by the fifth author, in which students match a list of course meta-objectives with the institution’s list of “Attributes of Graduates.” Especially during the first few weeks of the semester, we monitor the amount of scaffolding we provide to ensure that students remain challenged, but not overwhelmed. We also work hard to communicate effective messages throughout the course about the value of mistakes, growth mindset, and the importance of effort [7]. One instructor said,

Those that buy in do extremely well, while those that do not buy in will complain to no end on the [student evaluations]! To counter this I continually discussed the expectations and the results. After a test that had an average of around 68% I asked how everyone prepared. One person sheepishly raised his hand

and said he practiced the problems again and again. I thanked him and told the entire class he was one of the ones with an “A”. This reinforced the point that if they are putting in the work they will see the results. It was a turning point in the class and it could not have worked out better.

In the middle of the semester, we often use a program offered by the campus Center for Teaching and Learning (see Section 4.4) in which a faculty member from another division takes 20 minutes of class time to lead a “Helps / Hinders / Suggestions” focus group [12]. By vocally collecting this feedback from the whole class at once, students sometimes learn that an aspect of the course that they dislike is actually popular with other students. Finally, towards the end of the course we often assign students to write a 1-2 page reflection paper on what they have learned about themselves and the nature of mathematics. Through this process, students often realize that some of the hardest and most frustrating aspects of the course were the ones that resulted in significant, long-term learning.

3.3 Materials

Finding or creating good materials relevant to the particular teaching context is a standard roadblock for any instructor or program that seeks to use active learning, and even more so for inquiry-based learning [21] [24]. In all three of the initiatives described in Sections 5-7, we spent considerable time and effort in developing materials that would work for our students and instructors, in our context. While not including our materials directly in this paper, they are available at <https://www.usna.edu/MathDept/resources/calculus-projects.php>.

In each case, we found faculty eager to adopt these materials. On the other hand, we found the materials needed tweaking, even for the

same course in different semesters; “one size fits all” is not realistic. Our faculty survey indicates that many faculty prefer to write their own materials. One respondent said, “My experience is that, while it’s time consuming, if the instructor puts in the time to tailor the projects/create their own rather than just use the existing projects it tends to go better.”

3.4 Community

We have found that when groups of instructors implemented the ideas in professional development workshops together, rather than alone, the impact was magnified [13]. At the beginning of the CAPABLE project, the four developers each did different background reading and shared with each other what they had learned. In the project-based version of Calculus II, the four developers were also the four instructors using the materials, all for the first time; weekly meetings were incredibly valuable. Hearing about how each others’ students struggled or succeeded helped everyone develop more skills in understanding student thinking, and to provide the right amount of scaffolding to keep students engaged and challenged.

3.5 Seeds can grow

Each of the three initiatives we describe in Sections 5-7 involved about a year of very intense work. At the end of each, we had successes we were proud of. There were also issues that we knew were problematic; these were sometime discouraging, but we kept tinkering each semester and improving things. The intensity of the work died down for a time, until the next big project.

At the end of the CAPABLE project, the developers were ready to take a break and move on to other projects. We were uncertain whether anyone else would be motivated to use what we had developed.

But in fact, other instructors did continue to use the materials we had developed. The seeds had been planted, and our other two initiatives grew out of this work.

The Project Based Learning (PBL) Calculus II and IBL Calculus I projects have continued to thrive, even as different faculty members rotate in and out of teaching those courses. Some semesters there are more IBL sections, and some semesters there are fewer, but not zero. More generally, active learning has become the norm in our department.

4 DEPARTMENTAL SUPPORTS FOR LEARNING ABOUT TEACHING

Teaching is central to the mission of our institution, so our faculty tend to take it very seriously. Small class sizes and frequent class meetings ensure that we get to know our students well and become invested in their success. This common focus leads to a departmental culture that values time and effort spent on improving teaching. It is common in our department to discuss teaching informally in the hallways. In this setting, we celebrate each others' small successes, and group-source solutions to the dozens of little problems that arise throughout the semester [5]. In addition, our department has several more formal supports for instructor development, described in the rest of this section.

In preparation for this paper, we sent a short survey to the entire department. We borrowed some questions from Hayward and Laursen [22], and added the open-ended question, "How has your teaching changed, if at all, over the past ten years? What factors have contributed to your decision to change, or not change, aspects of your courses?" Our response rate was low, but included new and experienced faculty, permanent and three-year instructors, and faculty who favor traditional pedagogy, inquiry-based learning, and in between. All of those who responded

reported attending the Teaching Seminar, almost all reported participating in “Charm School,” and most reported attending either an event sponsored by the Center for Teaching and Learning, or an off-campus teaching workshop. Most of the quotes from instructors throughout this paper come from responses to this survey.

4.1 “Charm School”

Each fall our department holds a three-day new instructor training called “Charm School,” which is similar to the types of teacher preparation courses which are now common for graduate teaching assistants in mathematics Ph.D. programs. The bulk of Charm School is devoted to new faculty giving practice lessons to each other and to a group of experienced faculty volunteers, who give suggestions at the end of each lesson.

After the CAPABLE project started (see Section 5), a unit on active learning was added to Charm School. Last year the unit was expanded to include separate lessons on supervising group work and leading whole-class discussions. New instructors are now also given a copy of the MAA’s Instructional Practices guide [1].

4.2 Teaching seminar

In 2009, the department began holding a weekly teaching seminar to complement our colloquium and research seminars. Each week the seminar might be organized around an article, a course, a technology, or another topic of general interest. Over the years, the seminar has addressed a broad range of topics; active learning and IBL principles have been featured, but definitely not as the sole focus of the seminar series.

4.3 Course meetings

Course coordinators in the calculus sequence often hold instructor meetings, sometimes weekly, sometimes less frequently. These meetings often focus primarily on administrative concerns, but can also address course content or pedagogical ideas or concerns.

When two groups of instructors are using different methods, the question of whether to have combined or separate instructor meetings is still unsettled. The separate meeting is especially helpful for instructors using a method for the first time, since the group can collaboratively work out implementation details. On the other hand, separate meetings can leave instructors feeling somewhat like they are in separate camps. Collective meetings help everyone understand and agree on priorities, regardless of their teaching methods.

4.4 Center for Teaching and Learning

Our campus has a strong Center for Teaching and Learning that has been offering high quality workshops for many years. More recently, the Center for Teaching and Learning (CTL) has expanded its programs substantially to include, for example, an annual conference, as well as the midsemester formative teaching evaluation focus groups referenced in Section 3.2.

4.5 Summary

All of these supports have been important in removing common roadblocks to faculty adoption of active learning. For example, many new instructors have never experienced or observed an active learning classroom. Without this experience, they are less likely to try using it themselves [17]. Through Charm School, instructor meetings, peer observations, the Teaching Seminar, and workshops, we give all of our fac-

ulty opportunities to become more familiar with active learning and inquiry-based learning. Facilitating productive student discussions, presentations, or group work takes a different set of skills than delivering a polished lecture. Faculty in our department have used Charm School, the Teaching Seminar, instructor meetings, and informal mentoring to help build these skills.

Anecdotally, our department leadership has noticed that while many of the new instructors have first opted for more traditional teaching styles, they have converted in subsequent semesters to IBL methods once they see the benefits of increased engagement in their peers' IBL classrooms.

Nearly all common roadblocks become easier to overcome as more faculty adopt active learning in our department: there are more faculty mentors with active learning experience, there is more discussion of active learning in formal and informal settings, and our stockpile of active-learning materials targeting the needs of our students is growing. With more active learning practitioners in the department, there is also more wide-spread understanding of both the benefits and challenges of adopting active learning pedagogies, so the contributions of faculty who have adopted active learning are better understood when being evaluated for raises, promotions, and tenure.

The MAA's Characteristics of Successful Programs in College Calculus project [9] identifies seven characteristics of highly successful calculus programs. Five of these characteristics were present in our department ten years ago, and have been maintained or improved: a coordination system, with a common textbook and final exam and regular instructor meetings; a proactive set of academic and social support services (including the Academic Center); an effective placement system; challenging course content (all students take three semesters of calculus);

and robust teaching development systems (i.e., Charm School) [3] [9] [10]. The other two characteristics have grown in our department over the past 10 years. Overall use of student-centered pedagogies has increased; one instructor said “Active learning is now the norm in our department.” Reacting to local data in various forms has been key to the improvement and sustainability of our initiatives.

We shift focus now to describe three specific multi-instructor implementations of active learning at our institution, including obstacles and lessons learned.

5 CAPABLE (CALCULUS I, FALL 2009)

In 2009, a group of four faculty (including the first author) received summer funding from our campus to research, create, implement, and assess a “project-based” curriculum in Calculus 1. We called this effort “Calculus Acquisition through a Problem and Activity Based Learning Experience,” or CAPABLE.

5.1 Community, Support, and Coordination

All four developers of the CAPABLE project were permanent faculty with Ph.D.s in mathematical sciences who had been teaching on our campus for four years or more. A series of workshops on project-based learning offered by our campus Center for Teaching and Learning sparked our interest. We were particularly interested in the potential for project-based learning to improve student self-efficacy and attitudes towards mathematics and science, as well as their problem-solving abilities and habits of mind.

The four developers of the project had done significant informal study about teaching and learning, attending workshops on our campus and at national meetings, but none of us had formal training in education.

We started by doing some background reading (including [8], [11], [15], [31], [35]).

One of our early goals was to create something that would be used widely by other instructors in our department. Because of the coordinated nature of the course (see Section 3.1), and because this was new territory for us, we decided on an incremental approach, searching for projects that could be plugged in to one or two days of the existing course structure. Ideally, each project would prepare students for that day's homework problems. As we gathered materials (more about these in the next section), we identified a day in the course calendar that each project could be implemented.

Our goal of attracting instructors was successful: of the 20 instructors, 12 reported midsemester that they had used at least one of the projects linked from our website. Six instructors, including both new and more experienced faculty (and including two of the developers), used the projects routinely in their classes. One instructor who was teaching for the first time reported:

As a new instructor, the CAPABLE projects showed me that teaching is more than lecturing to a class. When I sit down to write a lesson plan, the first thing I do is go to the CAPABLE site for ideas. Students enjoy being actively involved during class and I appreciate the immediate feedback. Preparing an activity based class is more difficult than preparing a traditional lecture so it's helpful having access to material prepared (and tested) by experienced professors.

The four developers working on this project formed our own small community of practice, which went through the stages of development described by Wenger [39]; by 2010 we dispersed to teaching different

courses, taking with us the lessons we had learned and memories of this community.

5.2 Materials

We found that many projects for calculus classes already existed. The University of Delaware Problem-Based Learning Clearinghouse had a list of criteria for projects, that we modified based on our goals, situation, and what we had learned from our background reading [14]. We were in search of projects and activities that would:

- have a good “hook”
- involve realistic, familiar contexts when possible
- allow students to use and examine their prior knowledge
- include opportunities to emphasize metacognitive skills
- include both challenges, and scaffolding to allow students to succeed at those challenges
- push students to use higher-level thinking skills (e.g. on Bloom’s Taxonomy)
- include explicit reflection steps, on both the content and on the learning skills
- focus on meaning and communication.

We developed a list of about 60 class projects for first-semester calculus, each of which satisfied at least some of the above criteria. These were posted on a website available to all instructors [18]. About half of the projects were adapted from existing textbooks (especially [25], since republished as [26]). About a third of the projects listed were entirely new, written by ourselves, with the above criteria in mind.

5.3 Assessment

We were primarily interested in whether project-based learning would have an effect on students' self-efficacy and attitudes towards mathematics, their metacognitive skills, and ultimately on their choice of STEM or non-STEM major. To this end, we designed a survey (available on the CAPABLE website, [18]) and administered it during the first and last weeks of class. We also asked instructors to fill out weekly time logs classifying how they spent class time each day. Twelve of the 20 instructors filled out the logs for at least part of the semester.

Our initial assumption, based on the work of Laursen et. al. (see [27], [28]) was that instructors using our projects regularly would report more time spent on the three “active learning” categories (Class Discussion, Individual Activity, and Group Activity) than instructors not using the projects. But in fact we found no clear correlation. Some instructors never used our projects, but wrote their own activities; one instructor used all of our projects, yet had the highest reported amount of “Exposition”! Thus we were unable to meaningfully separate instructors into “control” and “treatment” groups. We were learning firsthand how messy research in mathematics education can be, and that documenting pedagogical strategies requires sensitivity and nuance.

We were able to use the results of the survey to learn some general information about our students, and how their attitudes changed (or did not) over the course of the semester. Most of this information was fairly depressing. We found that while most students agree that calculus relates to the real world, most do not themselves try to understand mathematics by relating it to their own experiences. Over the course of the semester, the percentage of students who agreed with “I do not expect to understand equations in an intuitive sense in this course; they must just be taken as givens,” and “Working on a homework problem for

more than a few minutes is just a waste of time” both went up; the number who agreed with “Mathematics can be beautiful as well as useful” went down. The one bright spot was an increase in the percentage of students who agreed with “I often draw pictures to help me understand new concepts.”

As noted above, our incremental approach was successful in attracting other instructors, giving them opportunities to practice the skills necessary to run an active learning classroom. This incremental approach was probably also one reason we had reasonable student buy-in. However, it placed severe constraints on the types of projects we could do: they had to fit within a class period and prepare students to do the homework problems in the textbook. This left us unable to do the kinds of in-depth, longer-arc projects we had initially envisioned, and that we felt would be beneficial to students in the long run. We made this the goal of our next initiative.

6 PBL / IBL CALCULUS II, FALL 2016

6.1 Community

A new community of practice around project-based learning and inquiry-based learning coalesced in the spring of 2016, when the first, third and fourth authors committed to build a Calculus II course. At this point, the third author had been teaching in our department for four years and the fourth author, a graduate of our institution, had been teaching with us for a year. The first author, meanwhile, had attended an additional four-day workshop on inquiry-based learning [2] and taught Calculus III using IBL for several semesters. The second author, a permanent faculty member with a Ph.D. in mathematics and a master’s degree in math education, joined the department just in time to participate in writing and implementing these projects.

Between the four of us, we taught 11 of the 17 sections of the course. The other sections were taught by experienced, highly skilled faculty members who did not feel the need for frequent meetings. This freed the four of us to spend an hour each week in detailed discussions of how to implement the projects. This regular collaboration amongst the four instructors proved to be the most valuable use of time as we proceeded.

In the spring of 2017, the same four authors again taught Calculus II, employing the same materials from the previous semester and leveraging the lessons learned from the fall. We were also joined by an instructor in his second semester of teaching. At the end of the semester, this instructor wrote, “this was about as challenging a teaching style as I could adopt. I have no idea if my students “did well” on the Final, however, I feel they are all much closer to academic independence today than they were before we started together this Spring semester on what was a learning experience for us both.”

Another new instructor adopted the materials and approach in the summer of 2017. Since then, there has been at least one instructor using this curriculum every semester. The development of materials and a calendar to help structure the implementation, as well as a supportive community of instructors using the same materials, seemed to address some of the barriers that in the past would have prevented faculty from taking up this active learning methodology [23].

6.2 Materials and Coordination

Our main goal for this project was again to create a set of materials that could be broadly used, and that would cover the entire semester’s curriculum. Based on the results of the CAPABLE project, we wanted longer projects that would encourage students to go deeper and develop their problem solving skills and habits of mind. We knew we needed to

address the balance between this depth and content coverage through careful attention to course goals and assessments. As a foundation, we settled on a set of materials developed by Mairead Greene and Paula Shorter at Rockhurst University for use in an IBL course [20]. We rearranged and modified these materials, and created new modules to fill the gaps between their curriculum and ours. Several of the new modules we wrote had a “hands-on” component: measuring the area of a shape cut out of foam core; playing with springs when talking about work. The modules we adapted and wrote can be found at <https://www.usna.edu/MathDept/resources/calculus-projects.php>.

Meanwhile the first author, as course coordinator, distilled a set of essential course learning outcomes, and listed these in a syllabus that was separate from the course calendar. Two course calendars, one “traditional” and the other “Projects,” were created, both of them covering these essential learning outcomes, in the same order, and each giving different suggestions of how to go beyond the essentials. Both calendars were given to instructors, who had the freedom to choose either one or to create their own. The final exam was written based on the learning outcomes listed in the syllabus.

6.3 Assessment, Support, and Student Buy-in

We compared the final exam performance of the two groups of students, and found that the average exam score of students in the project-based sections was 7.5% higher than the average score of students in the other sections. This variation may have many other explanations, but at least could be taken as evidence that no harm was being done. We also compared grades of the two groups of students in their Calculus III classes the following semester, and found no significant difference.

Though we did not receive summer funding to prepare for this project,

we did have other forms of institutional support. The provost had recently added “project-based learning” as a strategic initiative for the institution, and wrote us a letter of encouragement. Our chair was also supportive, assigning the first author to coordinate the course, and backing us up when students complained (see below).

Student response was mixed. One group was open and eager, for example the student who said:

The project-based setting of this class helped me understand deeper concepts behind the mathematics I am doing. I gained a much broader confidence in my skills and I know that I grew in my ability to think critically. Honestly, I looked forward to each class because I knew that 1) I would be exploring deeper concepts that related to earlier material and 2) I would be learning how to think through problems critically instead of memorizing formulas and standards.

Another group vehemently resisted, saying at the end of the course, “This way might work for some people, but I don’t learn math this way.” In the first two weeks of class, several students approached the department chair asking to be transferred from a project-based to a “traditional” section of the course. At our institution, section transfers are rare; the chair denied the requests, saying that the instructor’s pedagogical choices were not sufficient reason for a transfer. This decision caused some short-term pain, as those disgruntled students remained in their sections. However, the underlying message of support was a very important one: students were asked to give it a chance, and the instructors did not feel undermined by the chair. At the end of the semester, many students wrote comments like this one: “I hated it at first, but it seemed to get better over time. I do feel that I actually understand learning objectives rather than just knowing them well enough to repeat

them on a test and then forget.”

Overall, we got the sense that we were making progress on increasing the depth of student learning, but needed to work on helping students learn to learn this way.

7 IBL CALCULUS I, FALL 2018

7.1 Community, Support, Coordination, and Materials

As the team of Calculus II developers began to disperse, in the spring of 2018, the first and fifth authors gathered a team of instructors committed to teaching Calculus I with an inquiry-based curriculum in the fall. Three members of this team are permanent faculty members, who each have several years of college teaching experience, but specifically wanted to learn to teach using IBL. Two of them were able (with department funding) to attend a four-day IBL workshop over the summer [2]. Our chair also again assigned one of the development team to be the course coordinator. The team developed an IBL curriculum with four units, with the format alternating by unit between student presentations and group activities. Three of the units were sourced externally ([6], [19], [20]) and one, a special precalculus review unit, was developed by the first author.

The course coordinator followed our model that had been successful the previous year, of writing a syllabus with a list of essential course learning outcomes and separate “IBL” and “Textbook” course calendars which would each cover those outcomes. Both course calendars were set up so that one day each week was set aside for quizzes and review or catch-up, as needed, or extra material of the instructor’s choice. This “quiz day” helped keep all sections on about the same pace, and allowed students a day to review and assimilate, rather than pressing forward with new material every class.

In all, about half of all Calculus I teachers taught using the IBL curriculum, and about half used a standard curriculum on a similar schedule. The coordinator set up separate weekly course meetings for the two groups. Midsemester, instructors were paired up and observed each others' classes. This was a nice way to see what others were doing and get feedback. Several of the IBL Calculus I instructors went on to teach Calculus II in the spring, using the previously developed IBL materials. In the fall of 2019, more than half of the faculty teaching Calculus I used the IBL curriculum developed the previous year.

7.2 Assessment and Student Buy-in

Our main goal was to address the challenge of student buy-in, without losing the benefits we had seen in Calculus II. With this in mind, the list of learning outcomes included some meta-objectives; for example, “realize that mathematics is more about creating and deep thinking than memorizing procedures,” “gain social competence in negotiating different ways of thinking,” and “persevere when it gets difficult and frustrating” [37]. We highlighted the meta-objectives by setting aside time during the first several weeks to discuss them, and by incorporating them in a journal assignment early in the semester. The learning outcomes also placed a heavier than usual emphasis on concepts and reasoning, with around 20 proofs (such as the derivation of various derivative shortcut formulas) on the list of possible final exam questions. Many instructors used mastery grading, granting multiple opportunities to retake the formative assessments, including quizzes and writing assignments.

Student reactions were still mixed, but the negative reactions were much fewer and milder than in previous semesters. Students again made comments indicating they had learned deeply, such as, “Over the course of this semester my thinking has changed from a more static fact-based

understanding to a visual, dynamic knowledge of the concepts at work.”

The department’s assessment committee did an analysis comparing final exam scores between the IBL and textbook sections of the course. They found a 3% difference, with students in IBL sections earning higher scores on average, but this is potentially due to other factors, such as the relative experience level of the instructors in the two groups.

8 CONCLUSION

We hope that by describing our experiences in detail, we have provided both practical tips and general ideas for other departments and individuals. The successes we have had in our IBL initiatives would definitely not have been possible without mutual support. One faculty member said, “I have more value in the IBL based teaching style but find it hard to implement alone.” The small communities of practice we built around each of the three major initiatives have shifted and evolved over time. The department seems to have reached a new steady state, where traditional and inquiry-based sections are routinely working side by side. We are continually learning, supporting, and teaching each other.

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