Unique Pseudo-Expectations for C^*-Inclusions

Abstract: Pitts introduced *pseudo-expectations* in his 2011 GPOTS lecture at Arizona State, which I was fortunate to attend. A pseudo-expectation for the (unital) C^*-inclusion $\mathcal{A} \subseteq \mathcal{B}$ is a ucp (unital completely positive) map $\theta : \mathcal{B} \to I(\mathcal{A})$ such that $\theta(a) = a$ for all $a \in \mathcal{A}$. Here $I(\mathcal{A})$ is the *injective envelope* of $\mathcal{A}$. Every conditional expectation is a pseudo-expectation, but there are C^*-inclusions with no conditional expectations, while pseudo-expectations always exist by injectivity. In that same talk, Pitts announced that if $\mathcal{A} \subseteq \mathcal{B}$ is a regular MASA inclusion (for example if $\mathcal{A} \subseteq \mathcal{B}$ is Cartan), then there exists a unique pseudo-expectation. In these lecture I will survey 15 years of research on the *unique pseudo-expectation property*, in particular the (still elusive) search for a structural characterization that applies to all C^*-inclusions.