Topological orbit equivalence: an overview!

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In 1959, H.Dye introduced the notion of orbit equivalence and proved that any two ergodic finite measure preserving transformations on a Lebesgue space are orbit equivalent. He also conjectured that an arbitrary action of a discrete amenable group is orbit equivalent to a $\mathbb{Z}$-action. This conjecture was proved by Ornstein and Weiss and its most general case by Connes, Feldman and Weiss by establishing that an amenable non-singular countable equivalence relation$\mathcal{R}$ can be generated by a single transformation, or equivalently is hyperfinite, i.e., $\mathcal{R}$ is up to a null set, a countable increasing union of finite equivalence relations.

In 1992, using ideas of A. Vershik, Richard Herman, Ian Putnam and Christian Skau constructed a remarkable model for minimal homeomorphisms of the Cantor set ([HPS]). They associated to a Cantor minimal system (X, ϕ) an ordered Bratteli diagram and proved that the corresponding Bratteli-Vershik transformation is conjugate to ϕ. The approximately finite (AF) equivalence relation and the dimension groups associated to this model were key in the classification up to orbit equivalence of minimal free actions of $\mathbb{Z}$ ([GPS]) and later of $\mathbb{Z}^d$ ([GMPS1,2]). In this talk I will review these results of topological orbit equivalence.

References:

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