Abstract: In 1982, a visiting Israeli scientist at NIST named Dan Shechtman, made a discovery for which he was subsequently awarded the 2011 Chemistry Nobel Prize. While looking at X-ray diffraction patterns for a new Al Mn alloy, he observed Bragg peaks with fivefold symmetry. At the time, Bragg peaks were believed to imply a crystalline structure, but fivefold symmetry was, and still is, known to be impossible for crystals. “Impossible” alloys like the one Schechtman discovered came to be known as quasicrystals. Once controversial, they are now typically modeled by mathematical structures similar to Penrose tilings, with their X-ray diffraction patterns being modeled by Fourier analysis. We will start by giving a brief history of mathematical quasicrystallography, which involves subjects such as number theory, algebraic topology, dynamical systems theory, fractal geometry, and theoretical computer science. We will conclude by discussing some contemporary problems that arise in the Fourier theory of quasicrystals, and that go to the heart of the question of what should be considered to be a quasicrystal.