Norming for Discrete Crossed Products

A well-known result says that the operator norm of an $n \times n$ complex matrix can be computed by pre- and post-multiplying by unit vectors:

$$\|A\| = \sup\{|y^* A x| : x, y \in \mathrm{Ball}(\mathbb{C}^n)\}.$$

This formula admits a vast generalization: For any unital C^*-algebra $\mathcal{A}$ and any $X \in M_n(\mathcal{A})$,

$$\|X\| = \sup\{\|RXC\| : R \in \mathrm{Ball}(\mathrm{Row}_n(\mathcal{A})),\ C \in \mathrm{Ball}(\mathrm{Col}_n(\mathcal{A}))\}.$$

If $\mathcal{A} \subseteq \mathcal{B}$ is an inclusion of unital C^*-algebras, we say that $\mathcal{A}$ **norms** $\mathcal{B}$ provided the same formula holds for matrices over $\mathcal{B}$ (rather than just matrices over $\mathcal{A}$):

$$(\forall\ X \in M_n(\mathcal{B}))\ \|X\| = \sup\{\|RXC\| : R \in \mathrm{Ball}(\mathrm{Row}_n(\mathcal{A})),\ C \in \mathrm{Ball}(\mathrm{Col}_n(\mathcal{A}))\}.$$

Norming is a useful property for C^*-inclusions, since it allows one to deduce that certain bounded linear maps are actually completely bounded.

In these lectures, based on joint work with David Pitts (Nebraska) and Roger Smith (Texas A&M), we give two dynamical conditions on a discrete group action $G \curvearrowright \mathcal{A}$, each of which ensures that the C^*-inclusion $\mathcal{A} \subseteq \mathcal{A} \rtimes_r G$ is norming. Namely, either the action $G \curvearrowright \mathcal{A}$ is residually properly outer, or the induced action $G \curvearrowright \widehat{\mathcal{A}}$ on the spectrum is essentially free. In particular, if $\mathcal{A}$ is separable (resp. simple), then $\mathcal{A} \subseteq \mathcal{A} \rtimes_r G$ is norming provided $G \curvearrowright \mathcal{A}$ is properly outer (resp. outer).

The first lecture will be an introduction to norming for general C^*-inclusions, while the subsequent lecture(s) will establish the advertised norming results for discrete crossed products.