Chapter 2

Review of Intact Statical Stability

Learning Objectives:

1. Explain the concepts of righting arm and righting moment
2. Calculate the righting moment of a ship given the magnitude of the righting arm
3. Read, interpret, and sketch a Curve of Intact Statical Stability (or Righting Arm Curve).
4. Discuss what tenderness and stiffness mean with respect to naval engineering
5. Evaluate the stability of a ship in terms of:
   a. Range of stability
   b. Dynamic stability
   c. Maximum righting arm
   d. Maximum righting moment
   e. Angle at which the maximum righting moment occurs
6. Create a Curve of Intact Statical Stability for a ship at a given displacement and assumed vertical center of gravity, using the Cross Curves of Stability
7. Correct a GZ curve for a shift of the ship’s vertical center of gravity.
8. Correct a GZ curve for a shift of the ship’s transverse center of gravity.
9. Find the Metacentric Height by using the GZ curve
10. Analyze and discuss damage to ships, including
    a. Use added weight method to calculate ship trim, angle of list and draft
    b. Qualitatively discuss the lost buoyancy method
    c. List the Navy Damage Stability Criteria for ships
11. Analyze and discuss free surface effects, including:
    a. Consequences of free surface on overall ship stability
    b. Ways to limit the effects of free surface
    c. Calculate the effective metacentric height
d. Give the meaning of a negative metacentric height  

e. Correct the GZ curve for FSC

Before we dive into the complexity of ship performance in waves, let’s have a brief review of ship statical stability that you learned in EN342. This section is going to follow the course notes for Principles of Ship Performance (Chapter 4). That chapter is concerned with the ability of the ship to remain upright when external forces are trying to roll it over.

2.1 The Internal Moment for a Heeled Ship

As a ship heels over (due to an external moment), it develops an internal moment. If the ship is stable, the internal moment acts in the opposite rotational direction to the direction of the heel angle. In this situation, the resultant weight of the ship is often not in vertical alignment with the resultant buoyant force so internal moments are produced. Figure 2.1\(^1\) shows the sectional view of a ship that is being heeled over due to an external moment. It shows the relative positions of the center of gravity and the center of buoyancy for a ship that has been properly designed. Notice the perpendicular distance between the lines of action of the resultant weight and resultant buoyant force. This distance is the “righting arm” (GZ). To find the internal righting moment, multiply the righting arm by the magnitude of the resultant weight of the ship (or the magnitude of the resultant buoyant force). Thus, the righting moment is \(RM = GZ \cdot \Delta.\)

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\(^1\)from EN400 Course Notes
2.2 The Curve of Intact Statical Stability

Figure 2.1 is only a snapshot of the total stability picture. We are really interested in how the relationship between the buoyant force and weight changes as the ship heels over from zero degrees to large enough angles of heel to make the ship capsize. This is best represented with a graph of righting arm (GZ) versus heeling angle (\( \phi \)). The graph is called the “curve of intact statical stability” or the “righting arm curve”.

The curve of intact statical stability assumes the ship is being heeled over quasi-statically in calm water. Quasi-static means that the external moment heeling the ship over is doing so in infinitely small steps so that equilibrium is always present. Of course, this is impossible in reality, but it is acceptable as a concept for modeling the ship's stability. Be sure to realize that the predictions made by the curve of intact statical stability can not be directly applied to a rolling ship in a dynamic seaway (i.e. seakeeping). The dynamics of such a system, including the application of additional external forces and the presence of rotational momentum, are not considered in the intact statical stability curve. However, the intact statical stability curve is useful for comparison purposes. The stability characteristics of different hull shapes can be compared as well as differences in operating conditions for the same hull.

Figure 2.2\(^2\) shows a typical intact statical stability curve. When the ship is in equilibrium with no outside forces acting on it, the resultant weight of the ship will be vertically aligned with the resultant buoyant force. As an external moment heels the ship to port or starboard, the resultant weight and the resultant buoyant force will become out of vertical alignment, creating the righting arm. The righting arm will obtain a maximum value and then decrease until the resultant weight of the ship and the resultant buoyant force are again in vertical alignment. Heeling any further will cause the ship to capsize. Figure 2.3\(^2\) shows the vector diagrams of the buoyant and weight forces with respect to various points on the GZ curve in Figure 2.2.

Each intact statical stability curve is for a given displacement and given vertical center of gravity. The process of obtaining the actual intact statical stability curve is done by reading values off the “cross curves of stability” for a given displacement of the ship, and then making a sine correction to account for the proper vertical location of the center of gravity of the operating ship, a cosine correction for the correct transverse location of the center of gravity, and a sine correction to account for any free surface effect.

Several overall stability characteristics can be obtained from the curve of intact statical stability.

**Range of Stability**  This is the range of angles for which there exists a righting moment. The range starts at the angle corresponding to the ship’s equilibrium position with no external moments applied to it and goes to the angle at which the ship will capsize. For a ship with no initial angle of list the starting angle would be zero degrees. If the ship has a permanent angle of list, then the range is given from that angle of list to the capsizing angle of the heeled side. For example, in Figure 2.2, the range of stability is 0 to 85 degrees. The

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\(^2\)image from EN400 notes  
\(^3\)from EN400 course notes
greater the range of stability, the less likely the ship will capsize. If the ship is heeled to any angle in the range of stability, the ship will exhibit an internal righting moment that will right the ship if the external moment ceases (and no other dynamics are at play).

**Maximum Righting Arm** (GZ\textsubscript{max})  This is the largest internal moment arm created by the vertical mis-alignment of the buoyant force and the resultant weight vectors. It is simply measured as the peak of the curve of intact statical stability. In Figure 2.2 the maximum righting arm is 4.1 ft.

**Maximum Righting Moment**  This is the largest static moment the ship can produce. It is simply calculated from the product of the ship’s displacement (Δ\textsubscript{s}) by the maximum righting arm (GZ\textsubscript{max}). The standard units are in LT-ft. The larger the value of the maximum righting moment the less likely the ship will capsize. The maximum righting moment can’t be shown directly on the curve of statical stability. Only the maximum righting arm is shown.

**Angle of GZ\textsubscript{max}**  This is the angle of heel at which the maximum righting moment occurs. Beyond this angle the righting moment decreases to zero. In Figure 2.2 the angle of maximum GZ is 50 degrees. It is desirable to have this angle occur at large degrees of heel so that a rolling ship will experience a righting moment that increases in magnitude over a greater range of heeling angles.
Dynamic Stability  This is the work done by quasi-statically (very slowly) rolling the ship through its range of stability to the capsizing angle. Mathematically, this work is

\[
\Delta_s \int GZ d\phi
\]

This is the product of the ship’s displacement with the area under the curve of intact statical stability. The standard units are LT-ft. The dynamic stability can’t be shown directly on the curve of intact statical stability, but the area under the curve can be shown.

A Measure of the Tenderness or Stiffness  The initial slope of the intact statical stability curve indicates the rate at which a righting arm is developed as the ship is heeled over. If the initial slope is large, the righting arm develops rapidly as the ship is heeled over and the ship is said to be “stiff”. A stiff ship will have a short period of roll and react very strongly to external heeling moments. The ship will try to upright itself very quickly and forcefully. If the ship is too stiff, violent accelerations can damage ship structures and be harmful to personnel. If the initial slope is small, the righting arm develops slowly as the ship is heeled over and the ship is said to be “tender”. A tender ship will have a long
period of roll and react sluggishly to external heeling moments. Too tender of a ship can compromise stability and leave too little margin for capsizing. This initial slope is equal to the ship's metacentric height.

### 2.3 Cross Curves of Stability

The cross curves of stability are a series of curves on a single set of axes. The $x$-axis is the displacement of the ship in LT. The $y$-axis is the righting arm of the ship in feet. Each curve is for one angle of heel. Typically angles of heel are taken each 5 or 10 degrees. Figure 2.4 is a set of cross curves for the FFG-7.

The entire set of curves assumes an arbitrary location for the vertical center of gravity of the ship. Typically the assumed location of the center of gravity is the keel ($KG = 0$). The actual location of the assumed value of the center of gravity of the ship will always be marked on the cross curves. The cross curves are made by a series of integrations based on hull geometry. In summary, the intact statical stability curves, for a single displacement, comes from reading values off the cross curves of stability and correcting for the actual location of the center of gravity and any free surface effects that may be involved.

![Cross Curves of Stability](image)

Figure 2.4: Cross Curves of Stability for the FFG-7
2.4 Correcting for Center of Gravity Location

To correct the righting arm values from the cross curves of stability, the vertical and transverse position of the center of gravity must be known. The righting arm can be corrected for the vertical position of the center of gravity using a sine correction and the transverse position of the center of gravity using a cosine correction.

If $G_v$ is the final vertical location of the center of gravity, and $G_0$ is the initial location, then the value of the righting arm, $G_vZ_v$ at each angle of heel may be found using the following relationship:

$$G_vZ_v = G_0Z_0 - G_0G_v \sin \phi$$

where $\phi$ is the heel angle in question, $G_0G_v$ is the vertical distance between $G_0$ and $G_v$, and $G_0Z_0$ is the righting arm at the initial center of gravity location. If the center of gravity moves down, the distance $G_0G_v$ would be negative. If the hull were heeling to port, $\phi$ would be negative.

If $G_t$ is the final transverse location of the center of gravity, and $G_v$ is the initial location, then the value of $G_tZ_t$ at each angle of heel may be found using the following relationship:

$$G_tZ_t = G_vZ_v - G_vG_t \cos \phi$$

where $\phi$ is the heel angle in question, $G_vG_t$ is the transverse distance between $G_v$ and $G_t$, and $G_vZ_v$ is the righting arm at the initial center of gravity location. If the center of gravity moves to port, the distance $G_vG_t$ would be negative. If the hull were heeling to port, $\phi$ would be negative.

To combine into a single equation, consider a condition with an initial center of gravity position at $G_0$ (if starting at the cross curves of stability, this would be a center of gravity on the centerline at the keel). First we consider the vertical shift from $G_0$ to $G_v$ and then the transverse shift from $G_v$ to $G_t$:

$$GZ = G_0Z_0 - G_0G_v \sin \phi - G_vG_t \cos \phi.$$  

2.5 Free Surface Correction (at small angles of heel)

A free surface is fluid that is allowed to move freely, such as water in a partially filled tank. As the ship lists, the fluid in the tank moves. The fluid movement acts like a weight shift, causing the center of gravity of the fluid to move, which causes the ship’s center of gravity to shift in both the vertical and horizontal directions. The effect of the vertical shift is negligible at small angles ($\phi = 5$ to 7 degrees) but the transverse shift causes a decrease in the righting arm ($GZ$).

It is shown graphically in Figure 2.5\(^4\) that a vertical rise in the center of gravity also causes a shortened righting arm. The distance the center of gravity would have to rise to cause a reduction in the righting arm equivalent to that caused by the actual transverse shift is called the Free Surface Correction (FSC). The position of this new center of gravity is called the “virtual” center of gravity ($G_v$). The distance from the virtual center of gravity to the metacenter is called the Effective Metacentric Height ($GM_{eff}$).

\(^4\)from EN400 course notes
2.5.1 Static Effects

The static effects of a free surface are adverse resulting in a virtual rise in the center of gravity, a smaller range of stability, a smaller maximum righting arm, a smaller angle at which the maximum righting arm occurs, and an exaggerated list and trim if the ship is listing or trimming.

2.5.2 Dynamic Effects

It should be noted that the preceding analysis is referring to the static effects of a free surface. It has nothing to do with the dynamic effects of the water rushing back and forth. This effect is also detrimental, but is not described by the free surface correction. It is a common misconception to mix the dynamic effects of a free surface with the static analysis and the FSC. Baffles are a good way to minimize the dynamic effects of free surface.

2.5.3 Calculating the FSC and $GM_{eff}$

The free surface correction (FSC) created by a tank within a ship is given by the following equation:

$$FSC = \frac{\rho_i i_t}{\rho_s \nabla_s}$$

where $\rho_i$ is the density of the fluid in the tank, $\rho_s$ is the density of the water the ship is floating in, $\nabla_s$ is the underwater volume of the ship, and $i_t$ is the transverse second moment of area of the tank’s free surface area.

The formula for the second moment of area of a rectangle is given by the following equation:

$$i_t = \frac{(\text{length})(\text{width})^3}{12} = \frac{l \cdot b^3}{12}.$$
The free surface correction is applied to the original metacentric height to find the effective metacentric height:

\[ GM_{\text{eff}} = GM - FSC = KM - KG - FSC. \]

If any of the terms in this equation are not familiar to you, be sure to go back to your EN342 notes/book and review!

### 2.5.4 Effect of a Free Surface on GZ

A free surface causes a reduction in the ship's righting arm, range of stability, and dynamic stability. With a free surface, the ship now behaves as if the center of gravity were located at the virtual center of gravity. To calculate the effective righting arm of a ship with a free surface, the original righting arm must be corrected for the virtual rise in G caused by the free surface.

\[ G_1 Z_1 = GZ - FSC \sin \phi. \]

The worst case for a free surface is when the ship's transverse center of gravity is located off the centerline. Not only has the overall stability been reduced by the transverse location of G, but the effective rise in G due to the free surface further reduces the righting arm, range of stability, and dynamic stability. To correct the righting arm curve for a free surface and a transverse change in G, one must first correct GZ for the virtual rise in G caused by the free surface using the sine correction, then correct GZ for the transverse location of G using the cosine correction. This correction is given by the following equation:

\[ G_1 Z_1 = GZ - FSC \sin \phi - TCG \cos \phi. \]

### 2.5.5 Damage Stability

Naval ships are intended to go in harms way. When the shooting starts the object is to do harm to others, but sometimes damage to your ship is unavoidable. If the watertight portion of the hull is breached and water pours into the ship, the draft will increase, the trim will change, a permanent angle of list will result, and stability will be affected. In extreme circumstances the ship could be lost. This section discusses the fundamental behavior of a damaged ship and reviews 2 techniques that allow for its analysis:

- The Lost Buoyancy Method
- The Added Weight Method

**Lost Buoyancy Method**

In the lost buoyancy method we analyze changes in buoyancy rather than the center of gravity or displacement. Simply stated, the center of gravity remains the same (the ship weight, metal, etc. is constant) and any changes due to damage effect the distribution of the buoyancy volume. The total buoyant volume must remain constant since the weight of the
ship is not changing. The draft will increase and the ship will list and trim until the buoyant volume is regained.

The lost buoyancy method allows a damaged ship to be modeled mathematically so that the final drafts, list, and trim can be determined from assessed damage. The engineer can analyze every conceivable damage scenario and produce a damage stability handbook that may be used by the crew in the event of flooding. Using the lost buoyancy method allows “a priori” knowledge of the resulting stability condition of the ship so that appropriate procedures can be written and followed in the event of a breach in the ship’s hull.

**Added Weight Method**

As the name suggests, in the added weight method the ship is assumed undamaged but part of it is filled with the water the ship is floating in. This is equivalent to a weight addition. Provided the volume of the damaged compartment, its average location from the centerline, keel, and midships, and the water density is known, the change in the ship’s center of gravity can be predicted along with the consequences of this shift upon the draft, trim, and list of the ship.

**Permeability**  An added complication to the analysis of a damaged ship is the space available in a damaged compartment for the water to fill. When a compartment is flooded, it is rare for the total volume of this compartment to be completely filled with water. This is because the compartment will already contain certain equipment or stores depending upon its use. The ratio of the volume that can be occupied by water to the total gross volume is called the “permeability”:

\[
\text{Permeability} = \frac{\text{volume available for flooding}}{\text{total gross volume}} = \mu
\]

**US Navy Damage Stability Design Criteria**

There are quite a few Navy Intact Stability Design criteria, but here we review some of the damage criteria:

**Margin Line**  The margin line defines the highest permissible location on the side of the vessel of any damaged waterplane in the final condition of sinkage, trim, and heel. It is in no case permitted to be less than 3 inches (0.075 m) below the top of the bulkhead deck at the side.

**List**  The list caused by damage shall not exceed 20 degrees. This angle is too great for continuous operation of equipment. Naval machinery is designed to operate indefinitely at a permanent list of 15 degrees, although most equipment will probably remain functional up to about 25 degrees for at least a few hours. Personnel can continue damage control efforts effectively at a permanent list of 20 degrees. At a permanent list of 20 degrees, the ship will possess adequate stability against wind and waves to be towed, at the very least.
Extent of Damage to the Hull

1. Ships less than 100 ft long are required to withstand flooding in one compartment.

2. Ships 100-300 ft long are required to withstand flooding in any two adjacent compartments.

3. Warships, troop transports, and hospital ships over 300 ft long are required to withstand a hull opening of 15% of the length between perpendiculares.

4. Any other ship over 300 ft long is required to withstand a hull opening of 12.5% of the length between perpendiculares.
CHAPTER 2. REVIEW OF INTACT STATICAL STABILITY

Problems

1. Briefly describe why a ship displaying positive stability will return to a condition of static equilibrium after being subjected to an external upsetting moment. Use a diagram in your explanation.

2. A ship has a submerged volume of 112,000 ft$^3$ and a righting arm of 2 ft when heeling to 15 degrees. Calculate its righting moment when heeling at this angle.

3. Using the cross curves of stability provided for the FFG-7 in the notes, graph the Curve of Intact Static Stability for an FFG-7 at a displacement of 3500 LT with KG = 0 ft.

4. Plot a curve of intact statical stability for starboard heels only for a ship with the following overall stability characteristics:
   - Range of Stability: 0-90 degrees
   - Maximum Righting Arm: 3.8 ft
   - Angle of Maximum Righting Arm: 50 degrees
   - Righting Arm at 30 degrees of heel: 2 ft
   On the plot, sketch the curve of intact statical stability for a ship with a stiffer righting arm. Which ship is more stable?

5. Using the cross curves provided for the FFG-7, correct the Curve of Intact Statical Stability for the FFG-7 at a displacement of 4000 LT with a KG = 19 ft. Plot the curve. What is the maximum righting moment? What is the range of stability? What is the angle of GZ$_{max}$? What happens to the ship if a moment greater than the maximum righting moment is applied? What happens if the ship rolls to an angle greater than the range of stability?

6. A ship has a displacement of 7520 LT and KG = 23.5 ft on the centerline. At this condition the ship has the following characteristics:
   - Range of Stability: 0° to 85°
   - Maximum Righting Arm: 5.2 at a heeling angle of 50°
   What happens to the ship’s stability characteristics if the center of gravity is raised? What happens to the ship’s stability characteristics if the center of gravity is lowered? What happens to the ship’s stability characteristics if there is a change in the transverse location of G with no vertical change in G?

7. An FFG-7 class ship displacing 4000 LT has a KG = 18.5 ft and KM = 22.5 ft. There is a tank filled with fuel oil with fuel oil density of 1.600 slugs/ft$^3$ creating a free surface 32 ft wide and 50 ft long. The ship is floating in salt water with a density of 1.9905 slugs/ft$^3$. What is the effective metacentric height?