Chapter 7

Maneuvering Theory

Learning Objectives:

1. Explain the concepts of directional stability
2. Explain the relationship between controls-fixed straight line stability and the linearized equations of motion for maneuvering.
3. Describe the forces on a ship in a turn.
4. Describe the effect of the ship characteristics and rudder on turning ability.
5. Calculate the maneuvering hydrodynamic derivatives for a ship from experimental measurements.
6. Use the maneuvering hydrodynamic derivatives to determine the controls fixed straight line stability of a ship.
7. Use the maneuvering hydrodynamic derivatives to determine the steady turning radius of a ship.
8. Estimate appropriate rudder dimensions based on standard guidelines.
9. **Laboratory Objectives:**
   (a) Explain the process for determining the maneuvering hydrodynamic derivatives dependent on sway and yaw velocities and accelerations.
   (b) Describe the procedure for static and dynamic PMM testing and how that data is used to determine the hydrodynamic derivatives Y_v, N_v, $Y_{\dot{v}}$, $N_{\dot{v}}$, Y_r, N_r, $Y_{\dot{r}}$, and $N_{\dot{r}}$.

In this chapter we will go much more in-depth on the theory behind maneuvering, including the equations of motion and the hydrodynamic derivatives. We will discuss how to determine these derivatives experimentally and discuss how a ship turns in more detail. Finally, we will cover some rudder design considerations.

7.1 Elements of Controllability

1. **Coursekeeping** (or Steering) - The maintenance of a steady mean course or heading. Interest centers on the ease with which the ship can be held to the course.

2. **Maneuvering** - The controlled change in the direction of motion (turning or course changing). Interest centers on the ease with which change can be accomplished and the radius and distance required to accomplish the change.

3. **Speed Changing** - The controlled change in speed including stopping and backing. Interest centers on the ease, rapidity and distance covered in accomplishing changes.

Performance varies with water depth, channel restrictions, and hydrodynamic interference from nearby vessels or obstacles. Coursekeeping and maneuvering characteristics are particularly sensitive to ship trim. For conventional ships, the two qualities of coursekeeping and maneuvering may tend to work against each other: an easy turning ship may be difficult to keep on course whereas a ship which maintains course well may be hard to turn. Fortunately a practical compromise is nearly always possible.

Since controllability is so important, it is an essential consideration in the design of any floating structure. Controllability is, however, but one of many considerations facing the naval architect and involves compromises with other important characteristics. Some solutions are obtained through comparison with the characteristics of earlier successful designs. In other cases, experimental techniques, theoretical analyses, and rational design practices must all come into play to assure adequacy.

Three tasks are generally involved in producing a ship with good controllability:

1. Establishing realistic specifications and criteria for coursekeeping, maneuvering, and speed changing.

2. Designing the hull control surfaces, appendages, steering gear, and control systems to meet these requirements and predicting the resultant performance.

3. Conducting full-scale trials to measure performance for comparison with required criteria and predictions.

7.2 Basic Equations of Motion

For the axis fixed with respect to the Earth, the equations of motion for maneuvering are

$$
\begin{aligned}
X_0 &= m_\Delta \ddot{x}_{0G} && \text{Surge}\\
Y_0 &= m_\Delta \ddot{y}_{0G} && \text{Sway}\\
N &= I_z \ddot{\psi} && \text{Yaw}
\end{aligned}
$$

However, for convenience we want to discuss the ship forces and motions from the ship-fixed reference frame. To do that, we need to express the variables in the previous equations from the ship-fixed coordinate system rather than in the Earth reference frame.

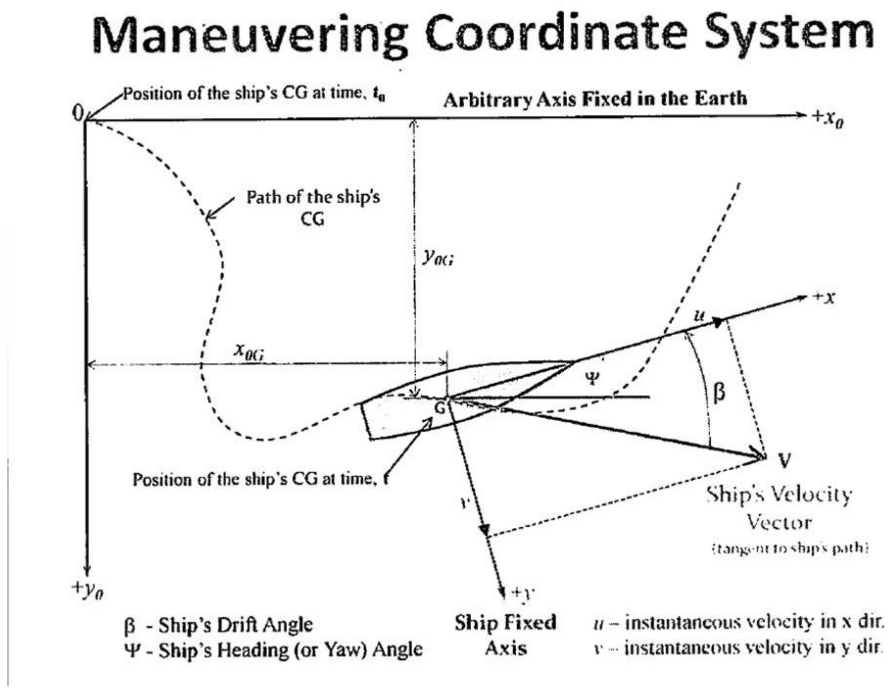

Figure 7.1: Coordinate System for Maneuvering (from reference 1)

Consider the velocities (draw a picture):

$$\dot{x}_{0G} = u\cos\psi + v\sin\psi$$
$$\dot{y}_{0G} = -u\sin\psi + v\cos\psi$$

To get accelerations we need to take the derivative of the velocities:

$$\ddot{x}_{0G} = \dot{u}\cos\psi + \dot{v}\sin\psi + (-u\sin\psi + v\cos\psi)\dot{\psi}$$
$$\ddot{y}_{0G} = -\dot{u}\sin\psi + \dot{v}\cos\psi - (u\cos\psi + v\sin\psi)\dot{\psi}$$

Plugging these into the equations of motion (*Note: the forces are still in the Earth reference frame*):

$$X_0 = m_\Delta(\dot{u}\cos\psi + \dot{v}\sin\psi + (-u\sin\psi + v\cos\psi)\dot{\psi})$$
$$Y_0 = m_\Delta(-\dot{u}\sin\psi + \dot{v}\cos\psi - (u\cos\psi + v\sin\psi)\dot{\psi})$$

Now consider the forces in the ship-fixed reference frame (same transformation as for the velocities):

$$X_0 = X\cos\psi + Y\sin\psi$$
$$Y_0 = -X\sin\psi + Y\cos\psi$$

Plugging into the previous equations and simplifying gives the equations of motion in the forces, velocities, and accelerations measured in the ship-fixed reference frame:

$$X = m_\Delta(\dot{u} + v\dot{\psi})$$
$$Y = m_\Delta(\dot{v} - u\dot{\psi})$$

The angular equation is unchanged by the shift in coordinate system. Since the other variables (u, v) are velocities, let's replace the angular velocity ($\dot{\psi}$) with r (now velocities have no dot and accelerations are all represented with one dot). Now, the equations of motion are:

$$\boxed{\begin{array}{c} X = m_{\Delta}(\dot{u} + vr) \\ Y = m_{\Delta}(\dot{v} - ur) \\ N = I_z \dot{r} \end{array}}$$

The forces and moments (left hand side) of the equations of motion consist of four types of forces that act on a ship during a maneuver:

1. Hydrodynamic forces acting on the hull and appendages due to ship velocity and acceleration, rudder deflection, and propeller rotation
   (a) Due to relative velocity and acceleration of the surrounding fluid
   (b) Due to rudder deflection
   (c) Due to propeller action
2. Inertial reaction forces caused by ship acceleration
   (a) Rigid body forces acting on a moving body - due to body accelerations
3. Environmental forces due to wind, waves and currents
4. External forces such as tugs, thrusters, mooring lines, etc.

We will only deal with the top two!

7.2.1 Linear Equations

The force components X, Y, and moment component N are assumed to be composed of several parts, some of which are functions of the velocities and accelerations of the ship. For now, we will assume that the forces are composed only of the forces and moments arising from motions of the ship which, in turn, have been excited by disturbances whose details we need not be concerned with here.

$$X = F_x(u, \dot{u}, v, \dot{v}, r, \dot{r})$$
$$Y = F_y(u, \dot{u}, v, \dot{v}, r, \dot{r})$$
$$N = F_\psi(u, \dot{u}, v, \dot{v}, r, \dot{r})$$

The forces are comprised of velocity dependent forces arising from hull drag through the water (in surge, sway and yaw) and acceleration dependent forces arising from the mass of the ship and the added mass of the fluid being accelerated in surge, sway, and yaw.

For stability analyses, we need to consider a vessel moving in equilibrium that experiences a disturbance. To consider the effect of a disturbance on the forces acting on the vessel, we can use the Taylor Series expansion technique. "If the function of a variable, x, and all its derivatives are continuous at a particular value of x, say x_1, then the value of the function at the value of x not far removed from x_1 can be expressed as follows":

$$f(x) = f(x_1) + (x - x_1)\frac{df(x_1)}{dx} + \frac{(x - x_1)^2}{2!}\frac{d^2 f(x_1)}{dx^2} + \frac{(x - x_1)^3}{3!}\frac{d^3 f(x_1)}{dx^3} + ...$$

If the change in the variable $(x - x_1)$ is kept small, the higher order terms (HOT) can be neglected, leaving

$$f(x) = f(x_1) + (x - x_1)\frac{df(x_1)}{dx}$$

For multivariable functions,

$$f(x, y) = f(x_1, y_1) + (x - x_1)\frac{\partial f(x_1, y_1)}{\partial x} + (y - y_1)\frac{\partial f(x_1, y_1)}{\partial y}$$

So, if we write the linearized sway force we get

$$Y = F_y(u_1, \dot{u}_1, v_1, \dot{v}_1, r_1, \dot{r}_1) + (u - u_1)\frac{\partial Y}{\partial u} + (v - v_1)\frac{\partial Y}{\partial v} + ... + (\dot{r} - \dot{r}_1)\frac{\partial Y}{\partial \dot{r}}$$

For *Straight Line Stability*, many of the velocities and accelerations are zero. For example, for a vessel moving at constant forward speed, there are no acceleration terms, no sway or yaw velocities and no Y force before a disturbance. The forward velocity is equal to the ship speed, U.

$$\begin{aligned}
&u_1 = U \\
&v_1 = r_1 = 0 \\
&\dot{u}_1 = \dot{v}_1 = \dot{r}_1 = 0 \\
&F_y(u_1, \dot{u}_1, v_1, \dot{v}_1, r_1, \dot{r}_1) = 0
\end{aligned}$$

Because of symmetry, there can be no Y force due to forward velocity or acceleration, so

$$\frac{\partial Y}{\partial u} = \frac{\partial Y}{\partial \dot{u}} = 0$$

The Sway Force Equation now becomes,

$$Y = \frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial \dot{v}}\dot{v} + \frac{\partial Y}{\partial r}r + \frac{\partial Y}{\partial \dot{r}}\dot{r}$$

We can perform the same technique to get the linearized Surge and Yaw equations:

$$\begin{aligned}
X &= \frac{\partial X}{\partial u}(u - U) + \frac{\partial X}{\partial \dot{u}}\dot{u} \\
N &= \frac{\partial N}{\partial v}v + \frac{\partial N}{\partial \dot{v}}\dot{v} + \frac{\partial N}{\partial r}r + \frac{\partial N}{\partial \dot{r}}\dot{r}
\end{aligned}$$

Now we have the forces for the basic equations of motion, we can combine (and move everything over to the right hand side) and get

$$
\begin{aligned}
0 &= m_\Delta \dot{u} + m_\Delta v r - \frac{\partial X}{\partial u}(u - U) - \frac{\partial X}{\partial \dot{u}}\dot{u} && \text{Surge} \\
0 &= m_\Delta \dot{v} - m_\Delta U r - \frac{\partial Y}{\partial v}v - \frac{\partial Y}{\partial \dot{v}}\dot{v} - \frac{\partial Y}{\partial r}r - \frac{\partial Y}{\partial \dot{r}}\dot{r} && \text{Sway} \\
0 &= I_z \dot{r} - \frac{\partial N}{\partial v}v - \frac{\partial N}{\partial \dot{v}}\dot{v} - \frac{\partial N}{\partial r}r - \frac{\partial N}{\partial \dot{r}}\dot{r} && \text{Yaw}
\end{aligned}
$$

The partial derivatives are called the *Hydrodynamic Derivatives* and we need to find them to solve the equations of motion!

7.2.2 Notes on Notation

To define a standard notation for maneuvering (rather than writing out the partial derivatives every time), SNAME (1952) specifed the following rule:

- Replace the partial derivative with the letter for force (or moment) and the subscript with the motion. For example,

$$
\begin{aligned}
\frac{\partial Y}{\partial v} &\equiv Y_v \\
\frac{\partial N}{\partial \dot{r}} &\equiv N_{\dot{r}}
\end{aligned}
$$

Rewriting the equations of motion using this notation gives the official Linearized Maneuvering Equations of Motion:

$$
\boxed{
\begin{aligned}
&- X_u(u - U) + (m_\Delta - X_{\dot{u}})\dot{u} + m_\Delta v r = 0 \\
&- Y_v v + (m_\Delta - Y_{\dot{v}})\dot{v} - (Y_r + m_\Delta U) r - Y_{\dot{r}}\dot{r} = 0 \\
&- N_v v - N_{\dot{v}}\dot{v} - N_r r + (I_z - N_{\dot{r}})\dot{r} = 0
\end{aligned}
}
$$

For convenience in analysis, we will non-dimensionalize the equations. For maneuvering the main effects are on sway and yaw - we can neglect surge since changes in forward velocity will be small relative to the mean forward velocity, U.

$$
\begin{aligned}
&- Y'_v v' + (m'_\Delta - Y'_{\dot{v}})\dot{v}' - (Y'_r + m'_\Delta) r' - Y'_{\dot{r}}\dot{r}' = 0 \\
&- N'_v v' - N'_{\dot{v}}\dot{v}' - N'_r r' + (I'_z - N'_{\dot{r}})\dot{r}' = 0
\end{aligned}
$$

(*The U disappeared in the sway equation since the velocities are non-dimensionalized by U, so $U' = 1$.*)

7.2.3 Control Forces and Moments

It is important to note that all the terms in the previous equations must include the effect of the ship's rudder held at zero degrees (on the centerline). On the other hand, if we want to consider the path of a ship with controls working, we must include terms expressing the control forces and moments created by rudder deflection (and any other control devices) as functions of time. The linearized y-component of the force created by rudder deflection is $Y_\delta \delta_R$. The linearized component of the moment created by rudder deflection about the z-axis of the ship is $N_\delta \delta_R$.

δ_R = rudder-deflection angle, measured from xz-plane of the ship to plane of the rudder; positive deflection corresponds to a turn to port for rudder(s) located at the stern.

Y_δ, N_δ = linearized derivatives of Y and N with respect to rudder-deflection angle δ_R

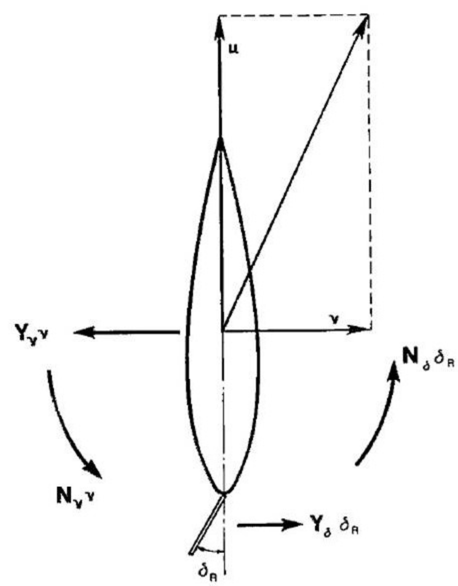

Figure 7.2: Rudder Induced Turning moments (from reference 1)

For small rudder deflections (due to small disturbances, for example) and for usual ship configurations,

$$Y'_{\dot{r}} \approx 0$$
$$N'_{\dot{v}} \approx 0$$

Applying these assumptions and including the rudder force and moment, the equations of motion become:

$$(I'_z - N_{\dot{r}})\dot{r}' - N'_v v' - N'_r r' = N'_\delta \delta_R \qquad \text{Yaw Moment}$$
$$(m'_\Delta - Y'_{\dot{v}})\dot{v}' - Y'_v v' - (Y'_r + m'_\Delta) r' = Y'_\delta \delta_R \qquad \text{Sway Force}$$

For conventional ship configurations, we can simplify the mass and inertial terms as follows:

$$(m'_\Delta - Y'_{\dot{v}}) \cong 2m'_\Delta$$
$$(I'_z - N'_{\dot{r}}) \cong 2I'_z$$

We can evaluate the hydrodynamic derivatives for the effect of the rudder on the hull, where δ_R is the rudder angle in radians (positive to **port**):

$$N'_\delta = \frac{\partial N}{\partial \delta_R}$$
$$Y'_\delta = \frac{\partial Y}{\partial \delta_R}$$

To make numerical predictions it is necessary to obtain values for some or all of the coefficients or derivatives involved. This is primarily done by means of captive model tests.

7.3 Captive Model Tests (PMM)

First let us consider what forces are acting on the vessel due to maneuvering motions and how these forces relate to the *Hydrodynamic Derivatives*.

Consider a ship experiencing transverse acceleration, $\dot{v}$ (see Figure 7.3). If the acceleration is to starboard (positive), there will be a reaction force $Y_{\dot{v}}$ to port due to the resistance of the water. For a transverse acceleration the force will always resist the direction of acceleration. This is shown in Figure 7.4 with the sway force versus sway acceleration showing a negative slope.

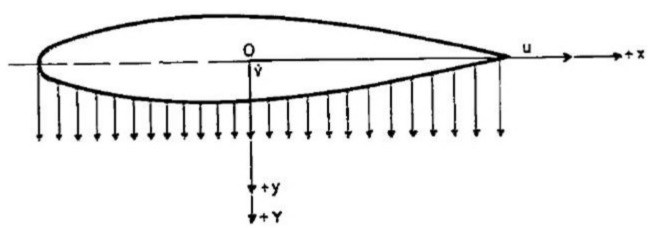

Figure 7.3: Ship Experiencing Transverse Acceleration

Consider a ship experiencing angular acceleration, $\dot{r}$ (see Figure 7.5). If the acceleration is positive (bow to starboard), there will be a reaction moment $N_{\dot{r}}$ in the negative direction due to the resistance of the water. For an angular acceleration the moment will always resist the direction of acceleration. Therefore, a plot of yaw moment versus yaw acceleration will always have a negative slope and will look like Figure 7.4.

Figure 7.6 shows the forces on a body with a sway velocity, v, added to a forward velocity, u. Both the bow and the stern experience a lift force oppositely directed to v. Therefore, Y_v is always negative (see Figure 7.7). However, the bow contribution is usually larger than that of the stern, so there is a negative moment contribution N_v. Yet the addition of a rudder at the stern will increase the magnitude of the stern force and so decrease the negative

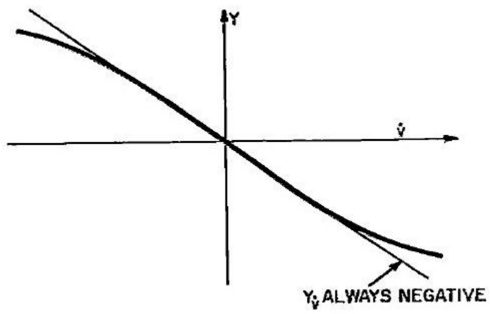

Figure 7.4: Sway Force versus Sway Acceleration

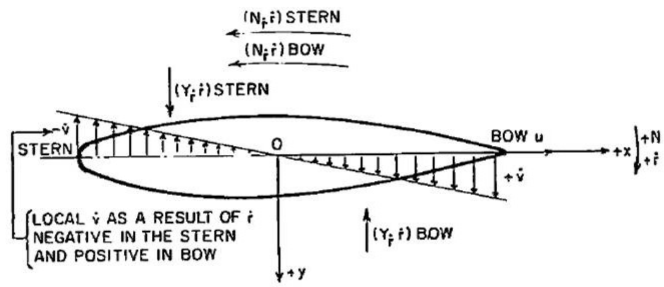

Figure 7.5: Ship Experiencing Angular Acceleration

magnitude of N_v. If the rudder force were sufficiently large, it might even cause N_v to be positive (not usually the case). Figure 7.7 show the possible relationships between N_v and v.

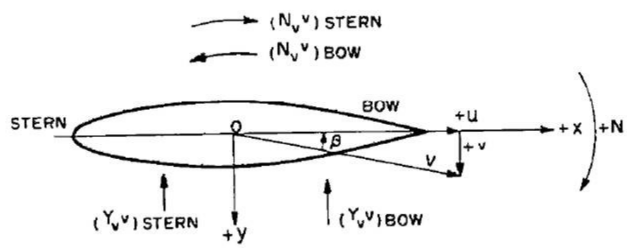

Figure 7.6: Ship Experiencing Forward Velocity and Transverse Velocity

Figure 7.8 shows the effect of an angular velocity, r, in addition to forward velocity, u, on Y and N. Due to the angular velocity, point B near the bow has a positive transverse velocity, v_B, producing a negative Y-force and a negative N-moment. Point S near the stern has a negative transverse velocity, v_S, producing a positive Y-force and a negative N-moment. So the moments can combine to produce a large negative moment, but the sway forces oppose each other resulting in a small positive or negative Y-force. Figure 7.9 shows the relationship between Y and N and r.

So, how can we use captive model tests and this information to find the hydrodynamic

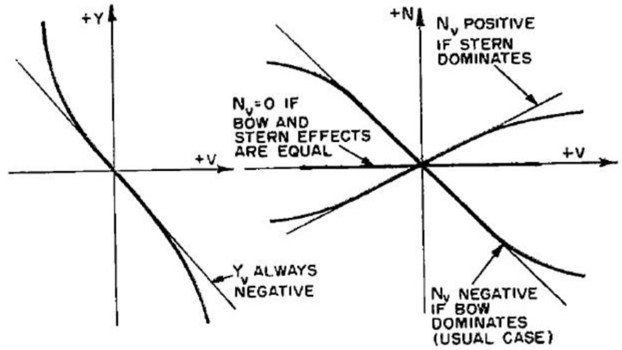

Figure 7.7: Sway Force and Yaw Moment versus Transverse Velocity

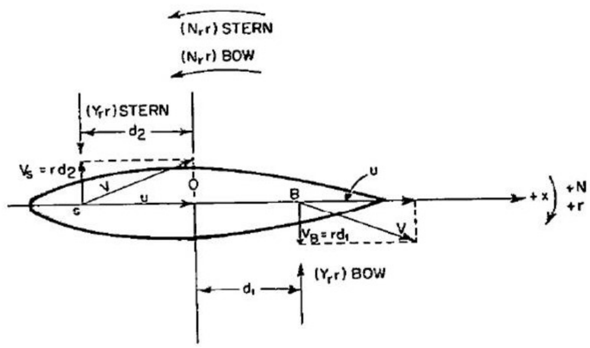

Figure 7.8: Ship Experiencing Forward Velocity and Angular Velocity

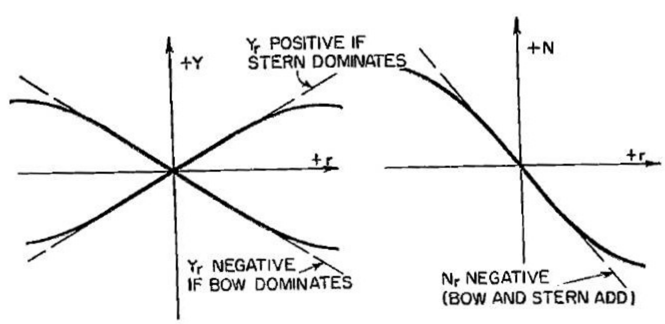

Figure 7.9: Sway Force and Yaw Moment versus Angular Velocity

derivatives?

7.3.1 Straight-Line Tests in a Towing Tank

The velocity-dependent derivatives Y_v and N_v of a ship at any draft and trim can be determined from measurements on a model of the ship, ballasted to a geometrically similar draft and trim, towed in a conventional towing tank at a constant velocity, U, corresponding to a given ship Froude number, at various angles of attack, β, to the model path. The figure below (Figure 7.10) shows the orientation of the model with respect to the tow tank. From the figure you can see that the transverse velocity component (from the vessel coordinate system) is produced along the y-axis such that

$$v = -U \sin \beta$$

where the negative sign is due to the sign convention (see Figure 7.1). The Y force and N

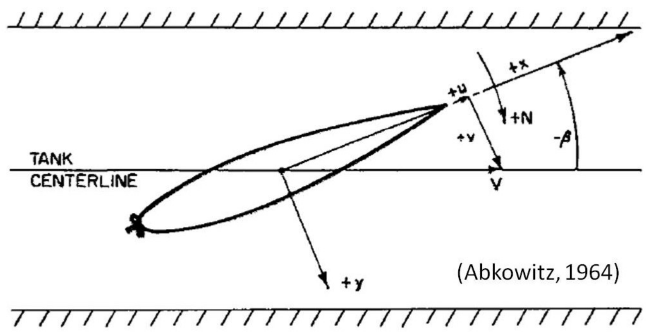

Figure 7.10: Straight Line Tow Testing

moment are measured on the model for each value of β tested. The force or moment versus sway velocity is then plotted and the hydrodynamic coefficient is the slope of the curve near $v = 0$. Figure 7.7 shows an example of sway force (Y) and yaw moment (N) versus sway velocity (v). The slope of the straight line through the curve at $v = 0$ is the hydrodynamic coefficient. So, for the plot Y versus v, you can find the coefficient Y_v and for the plot N versus v, you can find the coefficient N_v. Let's review:

1. Test a model fixed in yaw (specified drift angle, β) at a constant forward velocity, U.
2. The sway velocity felt by the model is equal to $-U \sin \beta$
3. The sway force and yaw moment are measured on the model
4. For a given U and β you have one point on the Y versus v plot and one point on the N versus v plot. To get additional points, run the test at various drift angles.

The propeller will usually exert an important influence on the hydrodynamic derivatives. Therefore, the model tests to determine these derivatives should be conducted with the propeller operating, preferably at the ship propulsion point. Also, since the undeflected rudder contributes significantly to the derivatives the model tests should also include the rudder in the amidships position.

The technique described above can also be used to determine the control derivatives Y_δ and N_δ. If the model is oriented with zero angle of attack ($\beta = 0$), but the model were towed down the tank at various values of rudder angle, δ_R, the force and moment measurements would determine the force Y and moment N as a function of rudder angle. Plots of these against rudder angle will indicate the values of the derivatives Y_δ and N_δ.

Straight-line tests can also be used to determine the cross-coupling effect of v on Y_δ and N_δ and of δ_R on Y_v and N_v.

7.3.2 Rotating-Arm Technique

To measure the rotating derivatives Y_r and N_r on a model a special type of towing tank and apparatus called a rotating-arm facility is occasionally employed.

- An angular velocity is imposed on the model by fixing it to the end of a radial arm and rotating the arm about a vertical axis fixed in the tank.
- The model revolves about the tank axis, rotates at rate r while its transverse velocity component v is zero at all times (yaw angle of attack or drift angle - $\beta = 0$).
- The model is rotated at a constant linear speed at various radii R and the sway force Y and yaw moment N are recorded.
- The angular velocity is given by $r = U/R$, so the only way to vary r at constant U is to vary R.
- The plots of Y and N versus r provide the hydrodynamic derivatives Y_r and N_r.

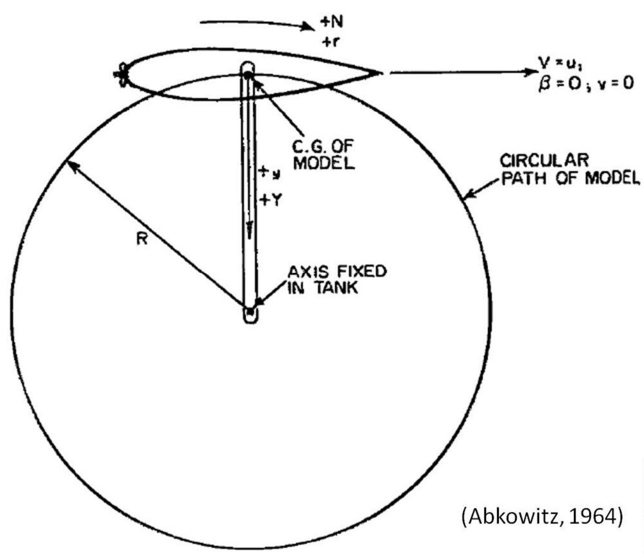

Figure 7.11: Model in Rotating-Arm Facility (from reference 1)

Some disadvantages of rotating-arm tests:

1. Require a specialized facility of substantial size. (There are only a few rotating-arm facilities in the world. One is at the David Taylor Research Center in Carderock, MD. Another was at the Davidson Laboratory at Stevens Institute of Technology.)
2. The model must be accelerated and data obtained within a single revolution. Otherwise the model will be running in its own wake and its velocity with respect to the fluid will not be accurately known.

3. In order to obtain values of the derivatives Y_r, N_r, Y_v, and N_v at $r = 0$, data at small values of r are necessary. This means that the ratio of the radius of turn, R to the model length L must be large.

7.3.3 Planar Motion Mechanism (PMM) Technique

To avoid the large expense of a rotating-arm facility, a device known as a Planar Motion Mechanism (PMM) was developed for use in the conventional long and narrow towing tank to measure the velocity-dependent and acceleration derivatives.

Basically the PMM consists of two oscillators, one of which produces a transverse oscillation at the bow and the other produces a transverse oscillation at the stern while the model moves down the towing tank at a constant forward velocity, U_0 (measured along the centerline of the tank). Figure 7.12 shows a sample model in a PMM. The forces required from each oscillator are recorded along with the transverse position of the model at each oscillator. The point B near the bow is oscillated transversely with a small amplitude, a_0,

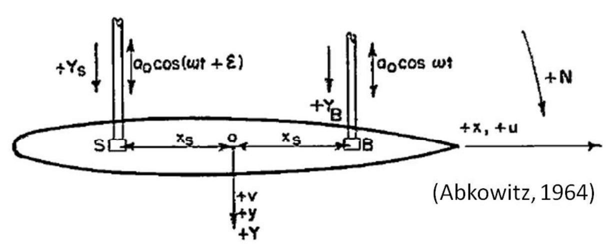

Figure 7.12: Model setup for planar motion tests

and at frequency ω. Point S near the stern is oscillated transversely with the same amplitude, a_0, and the same frequency, ω. The phase difference between the oscillations allows the model to experience yaw. If $\epsilon = 0$, the model experiences pure sway with zero yaw, as shown in Figure 7.13. For a pure sway test, the model is moving transversely in a sinusoidal

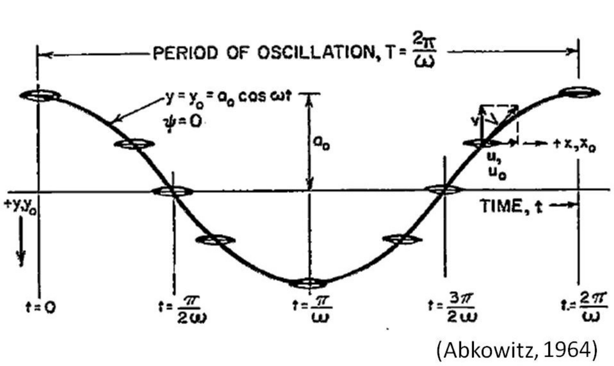

Figure 7.13: Path and orientation of model for pure sway motion

motion. The sway velocity and acceleration can be found by taking the time derivatives of the position.

$$
\begin{aligned}
y &= a_0 \sin \omega t \\
\frac{dy}{dt} &= v = \omega a_0 \cos \omega t \\
\frac{d^2 y}{dt^2} &= \dot{v} = -\omega^2 a_0 \sin \omega t
\end{aligned}
$$

Therefore, the magnitude of the velocity and acceleration is given by

$$
\begin{aligned}
v &= a_0 \omega \\
\dot{v} &= \omega^2 a_0
\end{aligned}
$$

Each oscillator measures the Y-forces experienced by the model as a result of the swaying motion (Y_B and Y_S). To find the Y_v derivative, we need to consider the Y-force in-phase with the velocity (or 90° out-of-phase with the position). To get the magnitude of the Y-force in-phase with the velocity we need to do a FFT of the signal (YIPPEE! I hear you cry $\ddot{\smile}$). This time, however, we will find the sine and cosine components of the signal, rather than the total magnitude. Once we have the components in-phase with the velocity (the cosine components) we can find the derivative Y_v by plotting the Y_{vel} term versus the sway velocity.

$$
Y_{vel} = Y_{B_{\cos}} + Y_{S_{\cos}}
$$

For the yaw moment derivative, a similar procedure can be applied. In this case, the sway force at each oscillator must be multiplied by a distance to get the moment. The distance, x_s, is typically chosen as measured from $\otimes$ (and each point B and S must be equidistant from $\otimes$). This means the hydrodynamic derivative N_v can be determined from plotting the cosine component of the yaw moment versus the sway velocity.

$$
N_{vel} = (Y_{B_{\cos}} - Y_{S_{\cos}}) x_s
$$

The components of the sway force and yaw moment that are in-phase with the acceleration are the sine components. Therefore, the derivatives $Y_{\dot{v}}$ and $N_{\dot{v}}$ are found by plotting the Y_{acc} and N_{acc} versus the sway acceleration $\dot{v}$.

$$
\begin{aligned}
Y_{acc} &= Y_{B_{\sin}} + Y_{S_{\sin}} \\
N_{acc} &= (Y_{B_{\sin}} - Y_{S_{\sin}}) x_s
\end{aligned}
$$

To obtain the angular derivatives Y_r and N_r from planar motion tests, the measurements must be made when $\dot{r} = 0$, $v = 0$, and $\dot{v} = 0$. Similarly, for $Y_{\dot{r}}$ and $N_{\dot{r}}$, the measurements need to be taken when $r = 0$, $v = 0$, and $\dot{v} = 0$. To impose an angular velocity and an angular acceleration on a body with v and $\dot{v}$ equal to zero, the model must be towed down the tank with the centerline of the model always tangent to its path, see Figure 7.14. This means the sway velocity (relative to the model) is always zero. To obtain pure yaw motion using the two oscillators in the PMM, the phase angle, ϵ, must be equal to

$$
\tan \epsilon / 2 = \frac{\omega x_s}{U}.
$$

(Abkowitz, 1954 and Gertler, 1959)

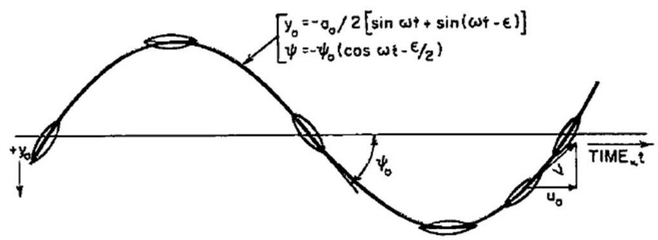

Figure 7.14: Path and orientation of model for pure yaw motion

The yaw oscillation is a sinusoidal motion and of the form

$$\begin{aligned}
\psi &= -\psi_0 \sin(\omega t - \epsilon/2)\\
r = \dot{\psi} &= -\omega\psi_0 \cos(\omega t - \epsilon/2)\\
\dot{r} = \ddot{\psi} &= \omega^2\psi_0 \sin(\omega t - \epsilon/2)
\end{aligned}$$

The yaw velocity, r is out-of-phase with the angle ψ and the angular accleration $\dot{r}$ is in-phase with the angle ψ. Therefore, the amplitudes of Y_B and Y_S measured 90° out-of-phase with ψ will determine the force and moment due to rotation r and the amplitudes of Y_B and Y_S in-phase with the ψ will determine the forces and moment due to angular acceleration $\dot{r}$.

$$\begin{aligned}
Y_{angvel} &= Y_{B_{\cos}} + Y_{S_{\cos}}\\
N_{angvel} &= (Y_{B_{\cos}} - Y_{S_{\cos}})x_s\\
Y_{angacc} &= Y_{B_{\sin}} + Y_{S_{\sin}}\\
N_{angacc} &= (Y_{B_{\sin}} - Y_{S_{\sin}})x_s
\end{aligned}$$

Plotting these forces versus velocity and acceleration can provide the yaw derivatives. The slope of Y_{angvel} versus r gives $(Y_r + m_\Delta U)$, the slope of N_{angvel} versus r gives N_r, the slope of Y_{angacc} versus $\dot{r}$ gives $Y_{\dot{r}}$, and the slope of N_{angacc} versus $\dot{r}$ gives $(N_{\dot{r}} - I_z)$.

7.4 Directional Stability

Now that we have experimental values for our hydrodynamic derivatives, we can solve the linear sway and yaw equations of motion. Solutions to the linear sway and yaw equations provide linear transfer functions permitting review of the stability of motion.

There are various kinds of motion stability associated with ships and they are classified by the attributes of their initial state of equilibrium that are retained in the final path of their centers of gravity. For example, consider Figure 7.15.

In each of the cases, the ship is assumed to be traveling at a constant speed along a straight path.

1. For case I – Straight Line Stability: the final path after the disturbance is finished retains the straight-line attribute of the initial state of equilibrium, but not its direction.

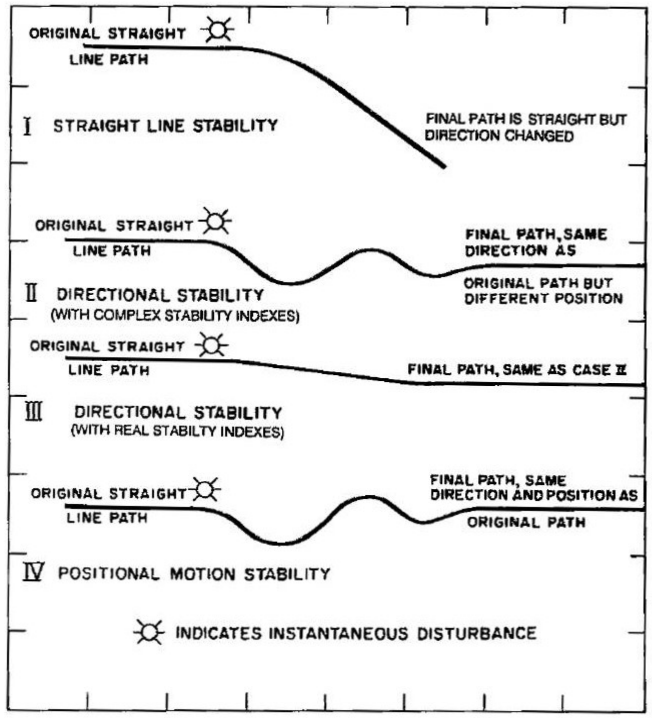

Figure 7.15: Various kinds of motion stability (PNA III, Arentzen 1960)

2. For case II – Directional Stability: the final path after the disturbance is finished retains not only the straight-line attribute of the initial path, but also its direction.

3. For case III - Directional Stability: the result is the same as for Case II, but without the oscillations.

4. For case IV – Positional Motion Stability: the ship returns to the original path - not only does the final path have the same direction as the original path, but also its same transverse position relative to the surface of the earth.

When operating with **controls-fixed** in the horizontal plane in the open ocean with stern propulsion, a surface ship does not have directional stability (i.e. if disturbed from its original course it will not return to that course by itself). However, the ship can have **Straight-Line Stability** (i.e. if disturbed from its original straight-line course, the ship will settle on a final path that is also a straight line).

When operating with **controls working** you can achieve directional stability. You want the ship to have directional stability with controls working, but also to have straight-line stability with controls fixed. This results in a compromise between rudder size and deadwood size.

We will start by using the ***linear equations of motion*** to evaluate the *straight-line stability* characteristics of a ship.

- We want to understand the effect of ship design features on maneuverability.

- With the rudder fixed on the centerline, we want the ship to have *straight-line stability*, but just barely.

- This will reduce the size of the rudder and steering gear needed for good maneuverability.

The simultaneous solution of the sway and yaw equations for the sway and yaw velocities yields a second-order differential equation. Working with non-dimensional variables, the solutions for v' and r' correspond to the standard solutions of second-order differential equations:

$$v' = V_1 e^{\sigma_1 t} + V_2 e^{\sigma_2 t}$$
$$r' = R_1 e^{\sigma_1 t} + R_2 e^{\sigma_2 t}$$

The variables V_1, V_2, R_1, and R_2 are constants of integration and σ_1 and σ_2 are the **stability indexes**. If both values of σ are negative, v' and r' will approach zero with increasing time which means that the path of the ship will eventually resume a new straight-line direction. If either σ_1 or σ_2 are positive, v' and r' will increase with increasing time and a straight-line path will never be resumed. We can relate these stability indexes, σ, to the hydrodynamic derivatives by substituting the solutions back into the equations of motion. If this is done, a quadratic equation in σ is obtained:

$$A\sigma^2 + B\sigma + C = 0$$

A, B, and C are as follows:

$$A = (Y'_{\dot{v}} - m'_\Delta)(N'_{\dot{r}} - I'_z) - Y'_{\dot{r}} N'_{\dot{v}}$$
$$B = Y'_v(N'_{\dot{r}} - I'_z) + N'r(Y'_{\dot{v}} - m'_\Delta) - N'_{\dot{v}}(Y'_r + m'_\Delta) - Y'_{\dot{r}} N'_v$$
$$C = Y'_v N'_r - N'_v(Y'_r + m'_\Delta)$$

The two roots, both of which must be negative for ***controls-fixed stability*** are:

$$\sigma_{1,2} = \frac{-B/A \pm [(B/A)^2 - 4C/A]^{1/2}}{2}$$

For both stability roots to be negative (all changes with respect to time are decreasing), two conditions must be met:

$$\frac{B}{A} > 0$$
$$\frac{C}{A} > 0$$

- For conventional ships A is *large* and *positive*.

- It can be shown that B is usually *large* and *positive* and on the same order of magnitude as A.

- This means that the **determining factor will be** C.

For both stability roots to be negative, $C > 0$! Therefore,

$$C = Y'_v N'_r - N'_v (Y'_r + m'_\Delta) > 0$$

Rewriting we can say,

$$\frac{N'_r}{Y'_r + m'_\Delta} - \frac{N'_v}{Y'_v} > 0.$$

We can calculate the directional straight-line stability after having performed the PMM tests on a model, but what can we say generally about controls-fixed straight-line stability from what we know about the hydrodynamic derivatives?

The terms N'_r and Y'_v are always negative, and generally large relative to Y'_r and N'_v. If the bow is dominate (the usual condition), Y'_r and N'_v are negative. So, in a conventional craft, the ration $\frac{N'_v}{Y'_v}$ will be small and since $\frac{N'_r}{Y'_r + m'_\Delta}$ is likely to be larger, the ship *will have* directional stability. For a conventional hull (where the bow dominates), directional stability can be increased by increasing the magnitude of Y'_v and N'_r. Adding a larger rudder in the stern of the ship increases the directional stability of the ship by decreasing the magnitudes of Y'_r and N'_v.

7.5 Analysis of Turning Ability

The response of the ship to deflection of the rudder, and the resulting forces and moments produced by the rudder, can be divided into 2 portions:

1. An initial transient one in which significant surge, sway and yaw accelerations occur

2. A steady turning portion in which rate of turn and forward speed are constant and the path of the ship is circular

Figure 7.16 shows the turning path of a ship. Generally, the turning path of a ship is characterized by four numerical measures: **advance**, **transfer**, **tactical diameter**, and **steady turning diameter**. All but the last are related to heading positions of the ship rather than tangents to the turning path. The *advance* is the distance from the origin at "execute" to the x-axis of the ship when that axis has turned 90°. The *transfer* is the distance from the original approach course to the origin of the ship when the x-axis has turned 90°. The *tactical diameter* is the distance from the approach course to the x-axis of the ship when that axis has turned 180°. These parameters of a ship's turning circle are useful for characterizing maneuvers in the open sea.

7.5.1 The Three Phases of a Turn

Phase I:

The first phase starts the instant the rudder begins to deflect and may be completed by the time the rudder reaches full deflection. The rudder force ($Y_\delta \delta_R$) and the rudder moment ($N_\delta \delta_R$) produce accelerations and are opposed solely by the inertial reaction of the ship (hydrodynamic responses have not yet materialized). For this phase the ship has not changed

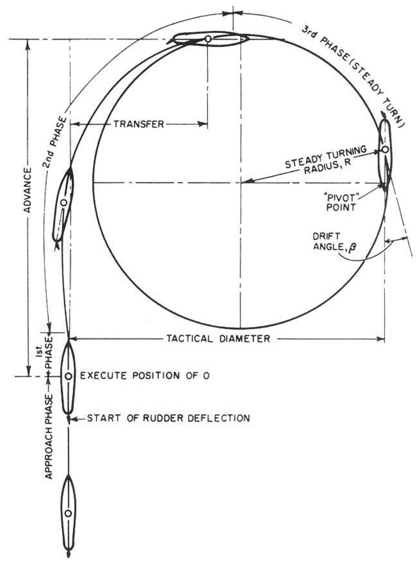

Figure 7.16: Turning Path of a Ship

direction, so $\beta = v/U = r = 0$. The linearized, dimensional equations for the first phase of turning are

$$(m_\Delta - Y_{\dot{v}})\dot{v} - Y_{\dot{r}}\dot{r} = Y_\delta \delta_R$$
$$(I_z - N_{\dot{r}})\dot{r} - N_{\dot{v}}\dot{v} = N_\delta \delta_R$$

These accelerations ($\dot{v}$ and $\dot{r}$) exist only for a moment, for they quickly give rise to a drift angle, β, and a rotation, r, of the ship.

Phase II:
The second phase starts with the introduction of the drift angle, β, and a rotation, r, of the ship. Here the accelerations of the ship *coexist* with the velocities and all the terms of the equations of motion along with the excitation terms $Y_\delta \delta_R$ and $N_\delta \delta_R$ are fully operative. The crucial event at the beginning of the second phase of the turn is the **creation of a $Y_v v$-force positively directed towards the center of the turn**. This force is introduced due to the drift angle, β. The magnitude of this force soon becomes larger than the $Y_\delta \delta_R$-force which is directed to the outside of the circle. The acceleration $\dot{v}$ ceases to grow to the outside of the circle and eventually becomes zero as the inwardly directed $Y_v v$-force comes into balance with the outwardly directed force of the ship. In the second phase of the turn, the path of the center of gravity of the ship at first responds to the $Y_\delta \delta_R$-force and tends towards the outside of the circle before the $Y_v v$-force grows large enough to enforce the inward turn.

Phase III:
Finally, after some oscillation (some of which is due to the settling down of the main propulsion machinery and is characteristic of the particular type of machinery and its control system) the second phase of turning ends with the establishment of the final equilibrium of forces. When this equilibrium is reached, the ship settles down to a turn of constant radius. This is the third, or steady, phase of the turn. In this phase v and r have non-zero values,

but $\dot{v}$ and $\dot{r}$ are zero. For this phase of the turn, the linearized equations of motion are:

$$-Y_v v - (Y_r + m_\Delta U)r = Y_\delta \delta_R$$
$$-N_v v - N_r r = N_\delta \delta_R$$

These two simultaneous equations can be solved for r and v provided that the stability derivatives (Y_v, Y_r, N_v, and N_r) and the control derivatives (Y_δ and N_δ) are known. Note that

$$r' = \frac{rL}{U} \qquad r = \frac{U}{R} \qquad r' = \frac{L}{R}$$

Solving the non-dimensional version of the linearized equations of motion shown above, we can solve for the turning radius, R, and the sway velocity, v':

$$\frac{R}{L} = -\frac{1}{\delta_R}[\frac{Y_v' N_r' - N_v'(Y_r' + m_\Delta')}{Y_v' N_\delta' - N_v' Y_\delta'}]$$

$$v' = -\beta = \delta_R[\frac{N_\delta'(Y_r' + m_\Delta') - Y_\delta' N_r'}{Y_v' N_r' - N_v'(Y_r' + m_\Delta')}]$$

A positive R denotes a starboard turn. The equation for the turn radius shows

- The steady turning radius is proportional to ship length and inversely proportional to rudder angle.
- Side velocity is equal to the drift angle and that is directly proportional to the rudder angle.
- Denominator in the equation for R introduces the effect of the rudder on the hull (N_δ' and Y_δ)
  - Sign of denominator is always positive
  - If the numerator is negative (straight-line stability) and the rudder is at the stern, a negative δ_R will give a positive R.

To **decrease the turning radius** we can:

1. Decrease Y_v' - could increase L/T ratio, but this is de-stabilizing
2. Generally increase N_v' (if N_v' is negative) - this is a result of different bow and stern shapes. Changes could be made by cutting away skeg and deadwood aft or increasing forefoot forward.
3. Increase N_δ' (obvious choice) - the trick is to do it without increasing $1/\delta$ too much. Want to move the rudder as far aft as possible and make the rudder as efficient as possible.
4. Increase Y_δ' (only if N_v' is negative) - can do this with a larger and/or more efficient rudder.

7.6 Rudder Design Considerations

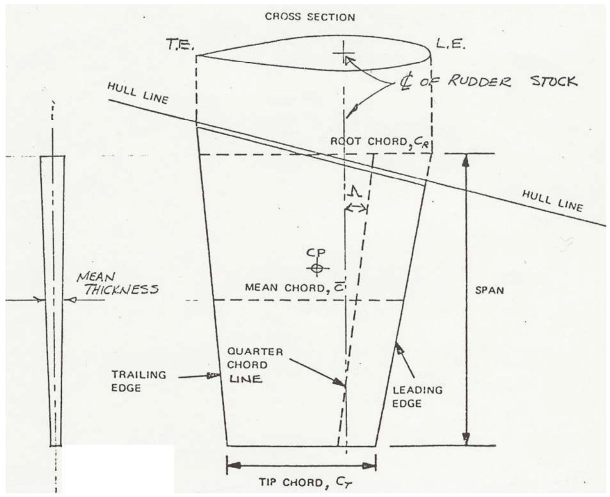

Figure 7.17: Rudder Definitions

7.6.1 Rudder Definitions

Figure 7.18 shows some important dimensions on a standard spade rudder.

- **Mean Span** - average of leading and trailing edge spans
- **Mean Chord** - average of the root and tip chords
- **Profile Area** - product of mean span and mean chord
- **Aspect Ratio** - ratio of mean span to the mean chord
- **Taper Ratio** - ratio of the tip chord to the root chord
- **Sweepback Angle** - angle between 1/4 chord line and vertical
- **Mean Thickness** - average of the max thickness of the foil at the root and tip

7.6.2 Lift, Drag and Angle of Attack

The lift (L) from an airfoil is defined as the component of force perpendicular to the free-stream velocity vector. The drag (D) from an airfoil is defined as the component of force parallel to the free-stream velocity.

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 c}$$
$$C_D = \frac{D}{\frac{1}{2}\rho U^2 c}$$

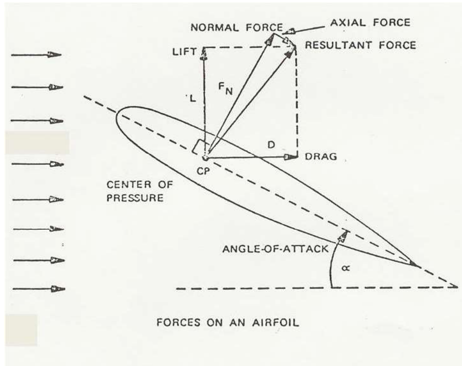

Figure 7.18: Forces on an Airfoil

The lift increases with increasing angle of attack. However, the lift cannot increase indefinitely with angle of attack. Eventually the adverse pressure gradient causes separation over the entire upper surface of the foil, resulting in a loss of lift. The maximum obtainable lift coefficient is called $C_{L,max}$.

- rudder stall often precedes a broach

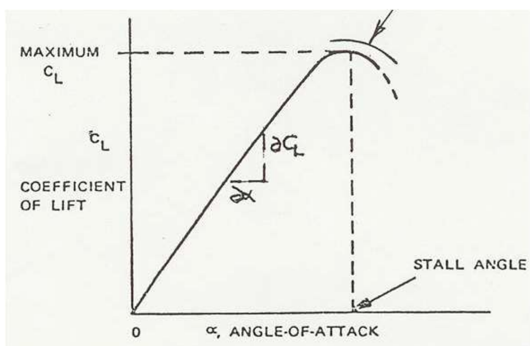

Figure 7.19: Lift Curve

7.6.3 Constraints on Rudder Design

In profile, the rudder needs to fit within dimensions dictated by the hull shape.

- The span is limited by the vessel draft.
  - It shouldn't extend below the baseline
  - It shouldn't penetrate the water surface
- The chord is limited by propeller clearance and stern shape.
  - The usual distance between the propeller and the rudder is 0.2·propeller diameter

The rudder should be designed for minimum drag at all speeds.

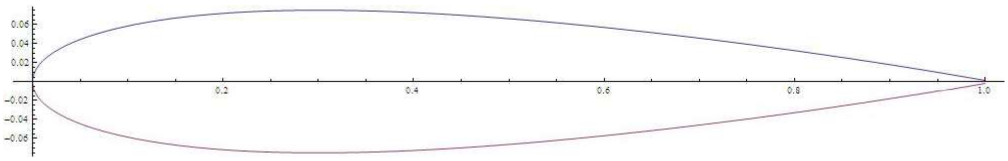

Figure 7.20: Typical Rudder Section

- The usual section shape is NACA 0015 (see Figure 7.20) to 0021 (relatively thick). These foils have a relatively constant center of pressure and thick sections are better structurally.
  - thickness-to-chord ratio is 0.15 to 0.21
  - symmetric shape
  - relatively low drag
  - max thickness at 30% chord length
- High aspect ratio
  - $\alpha =$ span/chord
  - very good lift-to-drag ratio

The rudder, rudder stock, rudder support and steering engine are considered together.

- Minimize size and weight of steering equipment
  - keep rudder weight as small as possible
  - keep torque on rudder stock as small as possible
    - balanced rudder - allows for smaller stock
    - semi-balanced rudder - support vs. moment
- Keep equipment as simple as possible
  - reduced repairs
  - simplifies layout

Undesirable effects of the rudder on the ship should be kept to a tolerable level (i.e. rudder induced vibration). From a hydrodynamic perspective, the basic considerations in rudder design can be summarized as follows:

- Full form ships need larger rudders
- Large rudders provide superior performance
- Put the rudder in the propeller wake to improve efficiency
- High aspect ratios give better efficiency

Table 31—General vessel hull form coefficients

Vessel Type	Typical Form Coefficients and Ratios C_B	L/B	B/T	Speed V, knots	Froude No. $V/\sqrt{gL}$	Number of Propellers/ Rudders	Rudder Area Ratios[a]	Dynamic Course Stability[b]
Harbor tug	0.50	3.3	2.1	10	0.25	1/1	0.025	S
Tuna seiner	0.50	5.5	2.4	16	0.31	1/1	0.025	S
Car ferry	0.55	5.1	4.5	20	0.34	2/2	0.020	S[e]
Container high speed	0.55	8.3	3.0	28.5	0.53	2/2	0.015	S
Container high speed	0.55	8.3	3.0	28.5	0.53	2/1	0.025	S[c]
Cargo liners	0.58	6.9	2.4	21	0.29	1/1	0.015	S
RO/RO	0.59	6.9	3.0	22	0.26	1/1	0.015	S
Barge carrier	0.64	7.5	2.9	19	0.20	1/1	0.015	S
Container Med. Speed	0.70	7.1	2.8	22	0.25	1/1	0.015	S
Offshore supply	0.71	4.7	2.75	13	0.28	2/2	0.016	S[d,e]
General cargo low speed	0.73	6.7	2.4	15	0.20	1/1	0.015	S
Lumber low speed	0.77	6.7	2.6	15	0.20	1/1	0.025	S
LNG (125 000 m^3)	0.78	6.8	3.7	20	0.20	1/1	0.015	U
OBO (Panamax)	0.82	7.5	2.4	16	0.17	1/1	0.018	U
OBO (150 000 dwt)	0.85	6.4	2.4	15	0.15	1/1	0.017	U
OBO (300 000 dwt)	0.84	6.0	2.5	15	0.14	1/1	0.015	U
Tanker (Panamax)	0.83	7.1	2.4	15	0.16	1/1	0.015	U
Tanker 100 000 to 350 000 dwt	0.84	6.2	2.4	16	0.15	1/1	0.015	U
Tanker 350 000 dwt	0.86	5.7	2.8	16	0.13	1/1	0.015	U
U.S. river towboat	0.65	3.5	4.5	10	0.25	2/2	...	U[d,e]

[a] Not for design guidance.
[b] U = unstable course stability; S = stable course stability.
[c] Although the vessel is directionally stable, maneuvering is difficult at low speeds when the propeller wash is not effective over the rudder.
[d] Maneuverability is good owing to installation of Kort nozzles, flanking rudders, and other capabilities.
[e] Twin screw because of restricted draft.

Figure 7.21: From PNA III (1989), p.346

  - limited by hull shape (span by draft; chord by stern shape)

- Rate of swing
  - increased rate of swing is good for small ships
  - large ships benefit more from rudder area than from swing rate

A good first estimate of rudder area can be achieved using the 1975 Det Norske Veritas (DNV) Rules.

$$A_R = \frac{T \times L_{BP}}{100}[1 + 25(\frac{B}{L_{BP}})^2] \tag{7.1}$$

The formula only applies for single rudders operating in a propeller wake. For all other arrangements DNV requires a 30% increase in area. (You want to put rudders behind propellers to increase the flow over the rudder at low speeds - makes the rudder more effective).

The equation gives (essentially) a rudder area coefficient. It is useful to compare values from the equation with values used in industry (see Figures 7.21 and 7.22). In choosing a design, the rudder performance is more affected by span length than chord length. An increase in the aspect ratio increases the lift/drag ratio.

Table 36—Rudder area coefficients

Vessel Type	Percent of $L \times T$
Single-screw vessels	1.6 to 1.9
Twin-screw vessels	1.5 to 2.1
Twin-screw vessels with two rudders (total area)	2.1
Tankers	1.3 to 1.9
Large passenger vessels	1.2 to 1.7
Fast passenger vessels for canals	1.8 to 2.0
Coastal vessels	2.3 to 3.3
Vessels with increased maneuverability	2.0 to 4.0
Fishing trawlers and vessels with limited sailing area	2.5 to 5.5
Seagoing tugs	3.0 to 6.0
Sailing vessels	2.0 to 3.0
Pilot vessels and farries	2.5 to 4.0
Motorboats	4.0 to 5.0
Keeled launches and yachts	5.0 to 12.0
Centerboard boats	30 or more

Figure 7.22: From PNA III (1989), p.371

Problems

1. State the linearized equations of motion for sway and yaw. Identify the hydrodynamic derivatives (and what they mean) and the vessel motion variables (and what they represent).

2. Explain what the hydrodynamic derivative $Y_{\dot{v}}$ represents, where it comes from, and what sign and relative magnitude it will be for ships of a conventional form. Provide a sketch!

3. Define the different types of directional stability for maneuvering.

4. Consider the results given below for a test model.

$Y'_v = -0.0105$	$Y'_{\dot{v}} = -0.00347$
$N'_v = -0.0012$	$N'_{\dot{v}} = -0.00002$
$Y'_r = -0.0008$	$Y'_{\dot{r}} + m = -0.0008$
$N'_r = -0.002$	$N'_{\dot{r}} + I_z = -0.00016$
$Y'_\delta = 0.0033$	$N'_\delta = -0.0017$

   (a) Determine whether the ship has controls fixed straight line stability.
   (b) Find the steady turning radius for this ship if the rudder angle is 15° starboard

5. What were the tests involved in the Static PMM and Dynamic PMM lab experiments? Describe the physical set-up, the inputs under our control, and the quantities which were measured. What results did these tests provide and why is this information useful for maneuvering analysis?

6. Describe the three phases of a ship's turn. Identify the start and end of each phase, the motions of the ship in yaw, sway, and surge, and the forces and moments acting on the ship.

7. State the basic considerations in rudder design from a hydrodynamic perspective. Give an equation for a first estimate of rudder area and state under what conditions this equation applies.

8. Consider a car ferry with a L/B ratio of 5.1. Find the rudder area as a percentage of the submerged lateral area ($L_{PP} \times T$).

9. The controllability of a Series 60 hull form was carried out using the PMM in the NAHL 380-ft towing tank. The following information was determined (scaled to ship dimensions):

Ship Information	**Hydrodynamic Stability Derivative**
$L_{WL} = 600$ ft	$Y'_v = -0.335$
$C_B = 0.70$	$N'_v = -0.097$
$B/D = 2.68$	$Y'_r = 0.075$
$m'_\Delta = 0.200$	$N'_r = -0.068$
	$Y'_\delta = 0.038$
	$N'_\delta = -0.019$

   (a) Does this ship have controls fixed straight line stability?
   (b) What would be the steady turning radius for this ship with a starboard rudder of 10°?
   (c) Which of the PMM tests can be done to solve for the Y'_v hydrodynamic derivative? *Select all that apply.*
      (i) Pure Yaw test
      (ii) Steady Drift Angle
      (iii) Pure Sway Test
      (iv) Steady Rudder Angle
   (d) Briefly describe the key elements of the three phases of a ship's turn. What terms are important in the equations of motion during each phase?

10. To solve the linearized equations of motion for a maneuvering ship, we need to determine the ten "hydrodynamic derivatives". One of these stability derivatives is N'_r.
   (a) What does this hydrodynamic derivative represent (in terms of motions of the ship and hydrodynamic forces)?
      (i) Sway force due to yaw moment
      (ii) Yaw moment due to yaw velocity
      (iii) Yaw moment due to sway velocity
      (iv) Yaw moment due to sway force
      (v) Sway force due to yaw velocity

(b) What is the sign (positive or negative) of this derivative? Provide a sketch.

(c) Why is the sign what it is? Is it always this way, or only for conventional displacement craft?

11. Sketch the motion of the model as it moves down the tank during the Dynamic Pure Yaw PMM Test.

(a) Select the inputs under the experimenters control. *Select all that apply.*

(i) carriage velocity
(ii) oscillation frequency
(iii) sway amplitude
(iv) sway forces
(v) surge amplitude
(vi) surge forces
(vii) propeller rpm
(viii) yaw amplitude

(b) Select the quantities measured directly. *Select all that apply.*

(i) carriage velocity
(ii) oscillation frequency
(iii) sway position
(iv) sway velocity
(v) sway acceleration
(vi) sway forces
(vii) propeller rpm
(viii) yaw position
(ix) yaw velocity
(x) yaw acceleration
(xi) surge forces

(c) What were the hydrodynamic stability derivatives determined from this part of the PMM experiment?

(i) Y'_v
(ii) $Y'_{\dot{v}}$
(iii) Y'_r
(iv) $Y'_{\dot{r}}$
(v) N'_v
(vi) $N'_{\dot{v}}$
(vii) N'_r
(viii) $N'_{\dot{r}}$
(ix) Y'_{δ_R}
(x) N'_{δ_R}