Chapter 18

Matlab Help

For those of you who may need a slight refresher in how to use MATLAB, have no fear! Here are some helpful tips for solving ODEs (like the kind found in problems involving simple harmonic motion) using MATLAB.

18.1 The Function File

To solve a problem like the oscillatory problems presented to you, you will need to solve a second-order ordinary differential equation. The equation you want to solve needs to be organized so that the acceleration ($\ddot{z}$) is on one side and all other terms on the other side of the equation so it can be put into a function file. A function file in MATLAB is a file that needs some inputs before it can calculate the outputs. A sample function file may look like

```matlab
function[sdot]=sample3(t,s)
y = s(1);
v = s(2);
ydot = v;
vdot = -2*v-4*y+3*sin(t);
sdot = [ydot,vdot];
```

or, more succinctly,

```matlab
function[sdot]=sample3(t,s)
sdot(1) = s(2);
sdot(2) = -2*s(2)-4*s(1)+3*sin(t);
% changes from a row vector to a column
sdot = sdot';
```

to solve an equation that looks like

$$\ddot{y} + 2\dot{y} + 4y = 3\sin(t).$$

It is important to understand that now your input “variable”, $s$, actually has TWO elements - the first element is the position ($y$) and the second element is the velocity ($v$). The function you wrote needs to provide the derivatives of BOTH input values. So, the output variable,
\textit{sdot}, also has \textbf{TWO elements -} the derivative of \textit{y} (velocity, \textit{v}) and the derivative of \textit{v} (acceleration, \textit{vdot}). KEEP IN MIND THAT \textit{s} AND \textit{sdot} \textbf{HAVE TWO VALUES -} \textit{s}(1) and \textit{s}(2) which correlates to \textit{sdot}(1) and \textit{sdot}(2).

\section{18.2 The Script File}

To solve a second-order differential equation like,

\[m \ddot{z} + b \dot{z} + cz = F_0 \sin \omega t\]

you need to use the \texttt{ODE45} (or \texttt{ODE23}) command. The command needs three inputs:

- The name of the function file (where you put the differential equation you want to solve)
- The time range you want to solve over and
- The initial conditions (generally the initial position and initial velocity of your mass).

Let’s use the example function file given above. The initial condition \textit{s0} must be a column vector containing one initial condition for each variable in the vector \textit{s}. For example, if the initial position \textit{y}_0 = 2 and the initial velocity \textit{v}_0 = \textit{v}_0 = 3, then you could type the following into the command window or a MATLAB script file:

\begin{verbatim}
% set the starting time 0 and the final time 3 (or however long we want)
tspan = [0,3];
% create a vector of initial conditions
s0 = [2,3];
% use ode45 to solve the ODE
[t,s] = ode45(@sample3,tspan,s0);
% the values for \textit{y} are in the first column of the result \textit{s}
y = s(:,1); % the values for the 2nd variable \textit{v} are in the 2nd column of \textit{s}
v = s(:,2);
% plot the position as a function of time
plot(t,y);
% label the axes
xlabel('t, in seconds');
ylabel('y, in whatever units');
\end{verbatim}
In this case, we could also use \texttt{plot(t,v)} to display the other variable (the second column of the output from \texttt{ode45}) which is the speed, as a function of time:

18.3 Multiple Line Plots

To plot more than one line on the same plot, you can use the \texttt{hold on} and write the plot command a second time (then be sure to use the \texttt{hold off} command). Or you can plot more than one line on the same figure within a single plot command. So, you should get the same plot with two lines on it if you typed,

\begin{verbatim}
plot(t,y,'b-')
hold on
plot(t,f,'r-')
hold off
\end{verbatim}

or

\begin{verbatim}
plot(t,y,'b-',t,f,'r-')
\end{verbatim}
For the forcing problem of a mass-spring system initially stretched and experiencing a forcing function, the equation of motion is re-written as two equations (based on \( v = \dot{x} \) and \( \dot{v} = \ddot{x} \)):

\[
v = \dot{x} \\
\dot{v} = \frac{F_0}{m} \sin(2t + \phi) - \frac{cx}{m}
\]

Let's say that \( F_0 = 10 \), \( m = 1 \), \( \omega_e = 2 \), \( \phi = \pi/2 \), and \( c = 5 \). These equations are then created into a function file in MATLAB with the inputs being \( t \) (time) and \( x \) (position) and the outputs being \( v \) (velocity) and \( \dot{v} \) (acceleration).

1. Open a **New Function File** and fill in the following:

```matlab
defunction [xdot] = SM1(t,x);
    omega0 = 2;
    m = 1;
    F0 = 10;
    phi0 = pi/2;
    c = 5;
    xdot(1) = x(2);
    xdot(2) = F0/m*sin(omega0*t+phi0) - c*x(1)/m;
    xdot = xdot';
end
```

2. Next we need to solve this ordinary differential equation so that we can get the position as a function of time as the output. MATLAB has several routines to do this for us, the most commonly used one called **ode45**. We are going to solve the function SM1.m over a time span of 0 to 10 seconds with initial conditions of \( x = 1/6 \) and \( \dot{x} = 0 \). The following lines can be simply typed into the command window, or written in a script file and saved.

```matlab
tspan = [0 10];
x0 = [1/6 0];
[t,x] = ode45(@SM1,tspan,x0);
position = x(:,1);
velocity = x(:,2);
plot(t,pos);xlabel('t, in seconds');
ylabel('x, in ft');```
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Problems

1. If an undamped spring-mass system with a mass that weighs 6 lbs and has a spring constant of 1 lb/in is suddenly set in motion at \( t = 0 \) by an external force of \( 4 \cos 7t \), draw a graph of the steady-state displacement versus time. Assume there is damping, but a negligible amount.

2. Consider a spring-mass-damper system. The weight of the system is assumed to be 220 lbs and the restoring force provides a spring constant of \( c = 68.5 \) lb/ft. Assume damping provides \( b = 22.4 \) lb-sec/ft of damping. The exciting force is assumed to have the form of a sine function with an amplitude of 224 lbs and a frequency of 3 rad/sec. What is the amplitude of motion? Determine (and discuss) the phase delay by plotting the excitation force on the same plot as the response. Any difference in the timing of the peak amplitudes indicates a phase difference (and the need for a phase angle in the equation for the motion response).

3. Using the system from problem 2, change the damping (first try increasing the value of \( b \), then decreasing it). What effect does this have on the amplitude of motion? Does it have any effect on the frequency of response? What effect does it have on the phase delay?

4. Using the system from problem 2, now change the mass (first try increasing the value of \( m \), then decreasing it). What effect does this have on the amplitude of motion? Does it have any effect on the frequency of the response? What effect does it have on the phase delay?

5. Using the system from problem 2, now change the stiffness (first try increasing the value of \( c \), then decreasing it). What effect does this have on the amplitude of motion? Does it have any effect on the frequency of the response? What effect does it have on the phase delay?

6. Discuss how this exercise relates to seakeeping analysis.