Chapter 7

Planar Motion Mechanism Experiment

Your team (4 or 5 people) have been asked to evaluate the directional stability of the FFG-7. In particular, you have been tasked to carry out appropriate experiments and calculations to determine:

1. the straight-line stability of the hull
2. the steady turning radius of the model for a rudder angle of 20°.

Your team will need to explain the details of the experiments carried out, explain the results from those experiments and relevant calculations, and complete the task assigned.

7.1 Background

Planar Motion Mechanism (PMM) tests provide a means of establishing numerical values for the hydrodynamic stability derivatives needed to quantify the directional stability of a particular model (and the parent ship). Various tests use one, two, or all forced displacements and motions to measure forces and moments. By appropriately choosing the tests to run, it is possible to find the hydrodynamic derivatives needed to solve for a vessel's straight-line stability or the vessel's controllability. To define all of the stability derivatives, we need to run a variety of tests. These tests fall into two categories: *Static Tests* and *Dynamic Tests*.

To determine the acceleration dependent coefficients for both sway and yaw, we need to perform two different *Dynamic Tests*. One test will be conducted so that the yaw velocity (r) and the yaw acceleration ($\dot{r}$) are held constant at a value of zero. This *Pure Swaying Mode Test* allows us to determine the sway velocity and acceleration dependent hydrodynamic coefficients (N'_v, Y'_v, $N'_{\dot{v}}$, and $Y'_{\dot{v}}$). The other test is the *Pure Yawing Mode Test*. In this test the sway velocity (v) and acceleration ($\dot{v}$) are held constant at a value of zero. This test allows us to determine the yaw velocity and acceleration dependent components (N'_r, Y'_r, $N'_{\dot{r}}$, and $Y'_{\dot{r}}$).

These tests include:

- *Static Tests*
  1. Drift Angle Test - provides Y'_v and N'_v
  2. Rudder Angle Test - provides the control derivatives Y'_δ and N'_δ
  3. Drift and Rudder Angle Test - cross-coupling effect of v on Y'_δ and N'_δ and δ on Y'_v and N'_v
- *Dynamic Tests*
  1. Pure Sway Test - provides both Y'_v and N_v and $Y'_{\dot{v}}$ and $N'_{\dot{v}}$
  2. Pure Yaw Test - provides both Y'_r and N_r and $Y'_{\dot{r}}$ and $N'_{\dot{r}}$
  3. Combination Test - provides nonlinear and cross-coupling terms

Because the model is forced to follow a given path with a specified speed and rudder angle during the tests, there is no real "equilibrium" operating condition being modeled in these tests. Instead, we need to vary all of the independent variables over their expected operating ranges systematically so that the equilibrium condition can be derived analytically from the cross-plotted experimental results. Because of the amount of computation required for these results, the signs of all the forces, moments and model movements must be meticulously defined and adhered to. The engineer's interpretation of the measured and analyzed signals is critical and depends on his or her understanding of the dynamics of what is going on in terms of the forces and moments during the model test. Figure 7.1 provides the sign convention used in these tests.

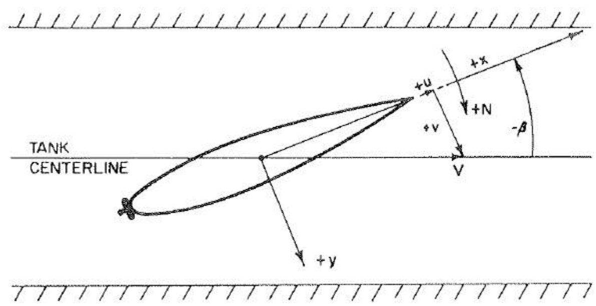

Fig. 31 Orientation of model in towing tank to determine Y and N (Abkowitz, 1964)

Figure 7.1: Sign Convention for Static PMM Tests

Note: the x_s in the notes refers to the distance from each post to the center of gravity. For this lab, $x_s = l_p/2$.

7.2 Static PMM Experiment

7.2.1 Static PMM PreLab

For the Static PMM tests:

1. Sketch and describe the sign conventions for the PMM tests (see Figure 7.1 in the lab assignment).
2. What are the Static PMM tests and which hydrodynamic derivatives can we find using it?
3. What are the parameters we set (or control) during the tests?
4. What is the data we collect?
5. What is the calculated data (results)?
6. Given the parameters we control and measure, write the equations to find: the sway velocity (v), the total surge force (X), the total sway force (Y), and the total yaw moment (N).
7. Write out the equations for non-dimensionalizing these variables.
8. What plots will you produce from these tests?
9. What result do we get from each plot? How do you calculate this result from the plot?

7.2.2 Static PMM Laboratory

Static Drift Test
The objectives of this experiment are:

1. Explain the process for determining the maneuvering hydrodynamic derivatives dependent on sway velocity and rudder angle.
2. Describe the procedure for static PMM testing and how that data is used to determine the hydrodynamic derivatives Y_v, N_v, Y_δ, and N_δ.

Static Drift Angle Test The model will be towed down the tank with a constant speed and with a fixed drift (or yaw) angle. The model must be towed at the self-propulsion point as determined from an SHP test. We control and monitor the following:

β - static drift angle	δ_R - rudder angle
U_c - carriage velocity (fps)	v - sway velocity $= -U_c \sin\beta$
n - propeller speed (rps)	l_p - distance between towing posts

During the test we will measure and record:

X_1 and X_2	forward and aft forces along the centerline, positive forward
Y_1 and Y_2	forward and aft forces perpendicular to the centerline, positive starboard
U_c	carriage velocity (ft/sec)
n	rotational speed of the propeller (rev/sec)

The tests will be run with $\delta_R = 0$ at a constant speed U_c with the β ranging from $-4°$ to $20°$ in steps of $2°$.

Static Rudder Angle Test The model will be towed down the tank with a drift angle of $0°$ and a constant velocity U_c. Again the model needs to be towed at the self-propulsion point. The rudder angle should be varied from $0°$ to $15°$ in steps of $2.5°$ to each side. For each test run X', Y', and N' are determined from the collected data. However, these values are now plotted on an abscissa of rudder angle, δ_R (radians). The slope of the curves for Y' and N' represent the rudder angle dependent stability derivatives Y'_δ and N'_δ.

Results – Tables and Plots
Prepare four plots:

1. Y' versus v'
2. N' versus v'
3. Y' versus δ_R
4. N' versus δ_R

Determine the slope of each curve (the straight-line portion near zero) and identify the experimentally determined values for Y'_v, N'_v, Y'_δ, and N'_δ. You will need to nondimensionalize the data using the following equations:

$$v' = \frac{v}{U}$$
$$X' = \frac{X}{1/2\rho U_c^2 L_{WL}^2}$$
$$Y' = \frac{Y}{1/2\rho U_c^2 L_{WL}^2}$$
$$N' = \frac{N}{1/2\rho U_c^2 L_{WL}^3}$$

Discussion of Results - required topics

1. Explain why the slope for each line is in the direction it is (positive or negative).
2. Is the relative magnitude of the slopes for each hydrodynamic derivative reasonable?

7.3 Dynamic PMM Experiment

7.3.1 Dynamic PMM PreLab

For the Dynamic PMM tests:

1. Sketch and describe the sign conventions for the PMM tests (see Figure 7.1 in the lab assignment).
2. What are the Dynamic PMM tests and which hydrodynamic derivatives can we find using it?
3. What are the parameters we set (or control) during the tests?
4. What is the data we collect?
5. What is the calculated data (results)?
6. Given the parameters we control and measure, write the equations or explain how to find: the total sway force (Y) in phase with the velocity, the total sway force (Y) in phase with the acceleration, the total yaw moment (N) in-phase with the velocity, and the total yaw moment (N) in-phase with the acceleration - *there will be a set of 4 equations for the Swaying Test and another set of 4 for the Yawing Test.*
7. Write out the equations for non-dimensionalizing these variables (Y'_{vel}, N'_{vel}, Y'_{acc}, N'_{acc}).
8. What plots will you produce from these tests and what result do we get from each plot? How do you calculate this result from the plot?
9. What are the equations for determining the directional stability of the model and the turning radius of the model given a rudder angle?

7.3.2 Dynamic PMM Laboratory

The independent variables under our control during both of these tests are:

ω - sway oscillation frequency (Hz) y_m - sway oscillation amplitude (ft)
U_c - carriage velocity (ft/s) n - propeller speed (rps)

Generally, we will be running all of the tests at the same carriage speed, chosen to represent a certain speed on the actual ship. The propeller speed is chosen so that the model is at a *self-propulsion point* based on the results of an SHP test. That leaves the sway amplitude and sway oscillation frequency to be varied on each test run to provide a range of sway and yaw velocities and accelerations.

Before each test run, a sway amplitude and sway acceleration combination need to be chosen. The PMM apparatus has practical limits in the range of values which can be used. The sway amplitude should be between 0.5 and 1.8 feet, and the sway frequency should be between 0.05 and 0.2 Hz.

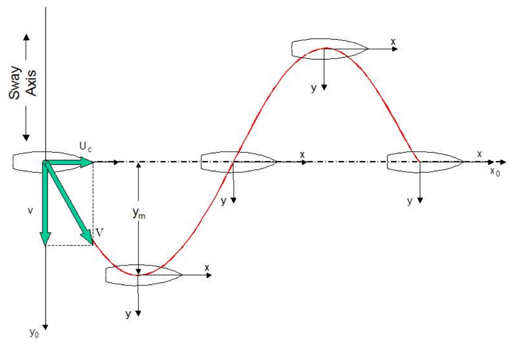

Figure 7.2: Pure Swaying Mode Test Conditions

During the test we will measure and record:

Y_1 and Y_2 - forward and aft forces perpendicular to the centerline, positive starboard
U_c - carriage velocity (ft/sec)
n - rotational speed of the propeller (rev/sec)
y - sway position (ft)
ψ - heading angle (degrees)

Truncate the data starting and ending at a zero up-crossing of the sway position, This insures an integer number of cycles and synchronizes the phase relationships needed to do the Fourier Transform.

As with the Static PMM, you will need to non-dimensionalize the motions and forces:

$$v' = \frac{v}{U} \qquad Y' = \frac{Y}{1/2\rho U_c^2 L_{WL}^2}$$

$$X' = \frac{X}{1/2\rho U_c^2 L_{WL}^2} \qquad N' = \frac{N}{1/2\rho U_c^2 L_{WL}^3}$$

Pure Swaying Mode Test

The model will be towed down the tank with a constant carriage speed (U_c) and with a fixed heading (or yaw) angle (ψ) equal to zero degrees. The model must be towed at the self-propulsion point as determined from an SHP test. The model is constrained so that the heading angle (ψ) is zero degrees (i.e. the local x-axis is parallel to the global x_0-axis. The model is oscillated back and forth along the sway axis by the sub-carriage assembly of the PMM. The amplitude of sway motion (y_m) and the frequency of the sway oscillation (ω) determine the magnitude of the sway velocity and acceleration. Figure 7.2 shows the layout for the pure swaying test condition. The carriage speed represents the x-component of the instantaneous velocity V. The y-component is the sway velocity, v. The sway velocity and sway acceleration ($\dot{v}$) are related to the sway amplitude (y_m) and the sway frequency (ω) as follows:

$$y = y_m \sin \omega t$$
$$v = \dot{y} = y_m \omega \cos \omega t$$
$$\dot{v} = \ddot{y} = -\omega^2 y_m \sin \omega t$$

We are interested in the maximum values of the sway velocity because the maximum velocity dependent force occurs at the maximum velocity, which is when the acceleration is zero. We are interested in the maximum values of the sway acceleration because the maximum acceleration dependent force occurs at the maximum acceleration, which is when the velocity is zero. The non-dimensional forms of sway velocity and acceleration are:

$$v' = \frac{y_m \omega}{U_c}$$

$$\dot{v}' = \frac{-y_m \omega^2 L_{WL}}{U_c^2}$$

By controlling the sway amplitude, sway frequency, and carriage speed, each test run will provide a different sway velocity and acceleration. We can then evaluate the measured force on each of the towing posts to determine the velocity-dependent and acceleration-dependent forces and moments.

In order to separate the force signal recorded on each post into velocity-dependent and acceleration-dependent forces, we will use a Fourier Transform. A MATLAB code to calculate the cosine and sine components of the forces will be provided.

The component forces need to be found for both the forward (1) and aft (2) force blocks. Once the in-phase and out-of-phase amplitudes have been found, the non-dimensional forces and moments can be determined. The velocity-dependent forces and moments are

$$Y'_{vel} = \frac{(Y_1)_{\cos} + (Y_2)_{\cos}}{\frac{1}{2}\rho L_{WL}^2 U_c^2}$$

$$N'_{vel} = \frac{[(Y_1)_{\cos} - (Y_2)_{\cos}] L_p/2}{\frac{1}{2}\rho L_{WL}^3 U_c^2}$$

while the acceleration-dependent components are

$$Y'_{acc} = \frac{(Y_1)_{\sin} + (Y_2)_{\sin}}{\frac{1}{2}\rho L_{WL}^2 U_c^2}$$

$$N'_{acc} = \frac{[(Y_1)_{\sin} - (Y_2)_{\sin}] L_p/2}{\frac{1}{2}\rho L_{WL}^3 U_c^2}$$

Four graphs need to be constructed in order to determine the hydrodynamic stability derivatives for this test. Graphs of N'_{vel} and Y'_{vel} versus v' and N'_{acc} and Y'_{acc} versus $\dot{v}'$ will allow us to find Y'_v, N'_v, $N'_{\dot{v}}$, and $Y'_{\dot{v}}$. The relationships are as follows:

$$Y'_v = \text{slope of } Y'_{vel} \text{ vs. } v'$$

$$N'_v = \text{slope of } N'_{vel} \text{ vs. } v'$$

$$(Y'_{\dot{v}} - m'_\Delta) = \text{slope of } Y'_{acc} \text{ vs. } \dot{v}'$$

$$N'_{\dot{v}} = \text{slope of } N'_{acc} \text{ vs. } \dot{v}'$$

Pure Yawing Mode Test

The model will be towed down the tank with a constant carriage speed (U_c) with a constantly varying heading angle (ψ) equal. This is accomplished by having the model follow a sinusoidal path with the centerline of the model always in-line with the tangent to the path. Again,

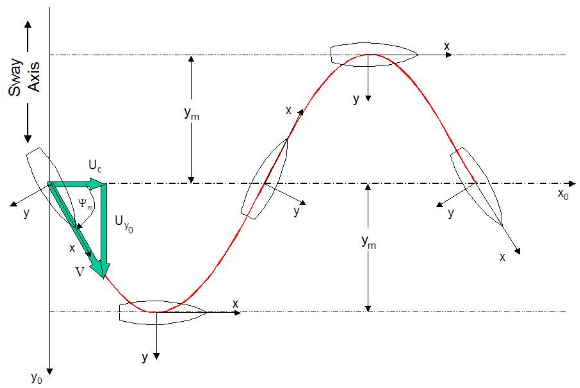

Figure 7.3: Pure Yawing Mode Test Conditions

the model must be towed at the self-propulsion point as determined from an SHP test. The model is constrained so that the drift angle (β) is always zero - the local x-axis is parallel to the instantaneous velocity vector (V). As in the Pure Sway Test, the model is oscillated back and forth along the sway axis by the sub-carriage assembly of the PMM. The amplitude of sway motion (y_M) and the frequency of the sway oscillation (ω) determine the magnitude of the sway velocity and acceleration. The difference now is that the model is also oscillating about a vertical axis through the center of gravity. Figure 7.3 shows the relationships of the key variables in he Pure Yawing Mode Test.

The motion of the model is determined by the two velocity components. The carriage is moving with a constant value (U_c) acting in the x_0-direction. The model is also traversing across the tank with a velocity (U_{y0}) which depends on the settings of sway amplitude (y_m) and sway frequency (ω). The resultant of these two velocity components is the instantaneous velocity (V).

Because the velocity component in the y_0-direction is varying with time, the instantaneous velocity will also vary with time. The trick is to have the bow of the model always pointing in the direction of V (so that $\beta = 0°$). This can be accomplished using the following relationship:

$$\tan\psi = \frac{U_{y0}}{U_c}$$

$$\psi = \tan^{-1}[\frac{y_m\omega}{U_c}\cos\omega t]$$

The significance of this is that the yaw motion is 90° out-of-phase with the sway motion.

By setting the maximum yaw amplitude to $\psi_{max} = y_m\omega/U_c$ and the frequency of the yaw oscillation to the same frequency as the sway oscillation, the desired path can be achieved.

We are interested in the maximum values of the yaw velocity and acceleration because the maximum velocity-dependent forces occur at the maximum velocity (where acceleration is zero) and the maximum acceleration-dependent forces occur at the maximum acceleration (where velocity is zero). The non-dimensional forms of sway velocity and acceleration are:

$$r' = \frac{\psi\omega_0 L_{WL}}{U_c}$$
$$\dot{r}' = \frac{-\psi\omega_0^2 L_{WL}^2}{U_c^2}$$

As with the Pure Sway Mode, we need to separate the force signals recorded on each post into velocity-dependent and acceleration-dependent forces. For the Pure Yaw Mode, the velocity dependent force will be out-of-phase with the yaw motion while the acceleration dependent forces will be in-phase with it. The component forces are from from a FFT of the force signal.

The component forces again need to be found for both the forward (1) and aft (2) force blocks. Once the in-phase and out-of-phase amplitudes have been found, the non-dimensional forces and moments can be determined. The velocity-dependent forces and moment are

$$Y'_{vel} = \frac{(Y_1)_{\cos} + (Y_2)_{\cos}}{\frac{1}{2}\rho L_{WL}^2 U_c^2}$$
$$N'_{vel} = \frac{[(Y_1)_{\cos} - (Y_2)_{\cos}]L_p/2}{\frac{1}{2}\rho L_{WL}^3 U_c^2}$$

while the acceleration-dependent components are

$$Y'_{acc} = \frac{(Y_1)_{\sin} + (Y_2)_{\sin}}{\frac{1}{2}\rho L_{WL}^2 U_c^2}$$
$$N'_{acc} = \frac{[(Y_1)_{\sin} - (Y_2)_{\sin}]L_p/2}{\frac{1}{2}\rho L_{WL}^3 U_c^2}$$

Four graphs need to be constructed in order to determine the hydrodynamic stability derivatives for this test. Graphs of N'_{vel} and Y'_{vel} versus r' and N'_{acc} and Y'_{acc} versus $\dot{r}'$ will allow us to find N'_r, Y'_r, $N'_{\dot{r}}$, and $Y'_{\dot{r}}$. The relationships are:

$$(Y'_r - m'_\Delta) = \text{slope of } Y'_{vel} \text{ vs. } r'$$
$$N'_r = \text{slope of } N'_{vel} \text{ vs. } r'$$
$$Y'_{\dot{r}} = \text{slope of } Y'_{acc} \text{ vs. } \dot{r}'$$
$$(N'_{\dot{r}} - I'_z) = \text{slope of } N'_{acc} \text{ vs. } \dot{r}'$$

Results - Tables and Plots

For the Pure Sway Test:

1. Y'_{vel} versus v'
2. N'_{vel} versus v'
3. Y'_{acc} versus $\dot{v}$
4. N'_{acc} versus $\dot{v}$

For the Pure Yaw Test:

1. Y'_{vel} versus r'
2. N'_{vel} versus r'
3. Y'_{acc} versus $\dot{r}$
4. N'_{acc} versus $\dot{r}$

Determine the slope of each curve (the straight-line portion near zero). Use these slopes and the relationships described in the lab to find Y'_v, $(Y'_{\dot{v}} - m'_\Delta)$, N'_v, $N'_{\dot{v}}$, $(Y'_r - m'_\Delta)$, $Y'_{\dot{r}}$, N'_r, and $(N'_{\dot{r}} - I'_z)$.

Discussion of Results - required topics

1. Calculate the A, B, and C values needed to determine the straight-line stability indices σ_1 and σ_2.
2. Evaluate the straight-line stability of the hull.
3. Determine the steady turning radius of the model for a specified rudder angle.

Deliverables - Discussion

1. Explain why the slope for each line is in the direction it is (positive or negative).
2. Are the results for straight-line stability and turning radius reasonable? Are these results desirable from a design perspective? Explain why or why not.