Roughness effects on wall-bounded turbulent flows

Karen A. Flack1 and Michael P. Schultz2
1Department of Mechanical Engineering, United States Naval Academy, Annapolis, Maryland 21402, USA
2Department of Naval Architecture and Ocean Engineering, United States Naval Academy, Annapolis, Maryland 21402, USA

(Received 31 May 2014; accepted 25 July 2014; published online 29 September 2014)

This paper outlines the authors’ experimental research in rough-wall-bounded turbulent flows that has spanned the past 15 years. The results show that, in general, roughness effects are confined to the inner layer. In accordance with Townsend’s Reynolds number similarity hypothesis, the outer layer is insensitive to surface condition except in the role it plays in setting the length and velocity scales for the outer flow. An exception to this can be two-dimensional roughness which has been observed in some cases to suffer roughness effects far from the wall. However, recent results indicate that similarity also holds for two-dimensional roughness provided the Reynolds number is large, and there is sufficient scale separation between the roughness length scale and the boundary layer thickness. The concept of similarity between smooth- and rough-wall flows is of great practical importance as most computational and analytical modeling tools rely on it either explicitly or implicitly in predicting flows over rough walls. Because of the observed similarity, the roughness function ($\Delta U^+$), or shift in the log layer, is a useful way of characterizing the roughness effect on the mean flow and the frictional drag. In the fully rough regime, it is shown that the hydraulic roughness length scale is related to the root-mean-square height ($k_{\text{rms}}$) and skewness ($\text{sk}$) of the surface elevation probability density function. On the other hand, the onset of roughness effects is seen to be associated with the largest surface features which are typified by the peak-to-trough height ($k_t$). Roughness function behavior in the transitionally rough regime varies significantly between roughness types. Since no “universal” roughness function exists, no single roughness length scale can characterize all roughness types in all the flow regimes. Despite this, research using roughness with a systematic variation in texture is ongoing in an effort to uncover surface parameters that lead to the variation in the frictional drag behavior witnessed in the transitionally rough regime. [http://dx.doi.org/10.1063/1.4896280]

I. INTRODUCTION

Over 80 years ago, Nikuradse1 investigated the effect of wall roughness on turbulent flows by measuring the pressure drop in pipes coated with uniform sand. His detailed experiments, insight into previous experiments, and correlation of his data set the stage for the prediction of rough-wall flows. This seminal work was extended by Colebrook2 to include commercial pipes and by Moody3 who consolidated the results into graphical form that could be readily used by practitioners.

The Moody3 diagram, shown in Figure 1, presents the friction factor, $f$, as a function of Reynolds number $Re_D = UD/\nu$ for a range of non-dimensional roughness heights, $k_s/D$, where $k_s$ is an equivalent sand roughness height, $U$ is the bulk mean velocity, $\nu$ is the kinematic viscosity of the fluid, and $D$ is the pipe inner diameter. The diagram represents three flow regimes: the hydraulically smooth regime where the wall shear stress is entirely due to fluid viscosity and $f = f (Re_D)$, the

---

1This paper was presented as an invited talk at the 66th Annual Meeting of the APS Division of Fluid Dynamics, 24–26 November 2013, Pittsburgh, PA, USA.
transitionally rough regime where the wall shear stress arises both from viscosity and pressure (or form) drag on the roughness elements and therefore, $f = f(Re_D, k_s/D)$, and the fully rough regime where the wall shear stress is due solely to form drag on the roughness elements such that $f = f(k_s/D)$.

While the Moody diagram has been and will continue to be an incredibly useful engineering tool for estimating the pressure losses in pipe flow, it has some significant practical limitations. Moody understood some of these issues and stated that he expected the friction factor obtained from the diagram to be accurate within about 10%. Seventy years later, with greatly improved measurement and computational tools, it is desirable to improve our predictive capabilities and fundamental understanding of rough-wall-bounded turbulent flows.

What then are the limitations of the Moody diagram? First, it is only strictly valid for surfaces in which the equivalent sand roughness height ($k_s$) (as shown on the diagram) is known a priori and that are operating in the fully rough regime. With regards to the first condition, $k_s$ is not a physical measure of the surface roughness but is instead the uniform sand roughness height from Nikuradse's experiments that produces the same friction factor as the surface of interest in the fully rough regime. Because of this, a hydrodynamic test in the fully rough regime is required to determine $k_s$ for a generic roughness before its skin-friction can be predicted. Additional tests have been performed for a range of rough surfaces as listed on the Moody diagram and published by numerous other researchers. However, one should be cautioned that a measured $k_s$ is only valid for the tested surface. Variation in surface texture arising from differences in a wide range of factors such as manufacturing, wear, corrosion, surface preparation, coating application, and fouling of the surface can significantly alter the equivalent sand roughness height for a surface. Ideally, the roughness length scale used in a predictive tool should be directly related to the texture of the surface itself.

The Moody diagram implies that all roughness types share similar skin-friction behavior in the transitionally rough regime. The shape of the Moody diagram friction curves are based on the function proposed by Colebrook. Guided by tests of commercial pipes (galvanized iron, wrought iron, and tar-coated cast iron), Colebrook's function for the transitionally rough regime shows a monotonic variation in the skin friction which asymptotically approaches the hydraulically smooth condition at low Reynolds number and the fully rough state at high Reynolds number. This stands in contrast to the transitional behavior of Nikuradse's uniform sand which departs from the hydraulically smooth condition abruptly and shows inflectional behavior. Colebrook conjectured that Nikuradse’s results may not have been indicative of the behavior of naturally occurring, non-uniform roughness due to
the monodispersity and close-packed nature of his sand-roughened surfaces. Interestingly, this has not proven to be the case. More recent results indicate that many uniform and non-uniform rough surfaces do not follow the Colebrook function in the transitionally rough regime. For example, honed surfaces (Refs. 4 and 5), commercial steel (Ref. 6), sandpaper (Ref. 7), and painted and sanded surfaces (Ref. 8) all show abrupt departure from the hydraulically smooth regime with many also displaying inflectional behavior in the transitionally rough regime.

To this point, the topic of roughness effects has centered on the Moody diagram and turbulent pipe flows. However, since roughness can influence the skin friction in a range of internal (e.g., pipes and ducts) and external (e.g., boundary layers) flow types, it is useful to introduce a parameter that allows comparison among disparate flow geometries. This parameter is termed the roughness function ($\Delta U^+$). Clauser$^9$ and Hama$^{10}$ each independently (and nearly simultaneously) introduced the roughness function concept. Both Clauser and Hama observed that primary effect of surface roughness on the mean velocity profile was to generate a downward shift in the log law indicative of a rise in momentum deficit compared to the smooth-wall case. This downward shift was $\Delta U^+$. They noted, however, that shape of the mean velocity profile in the overlap and outer layer was unaffected by the roughness. For this reason, the log law for rough walls can be expressed as Eq. (1), where $\kappa$ is the von Karman constant, and $B$ is the smooth-wall intercept.

$$U^+ = \frac{1}{\kappa} \ln y^+ + B - \Delta U^+. \quad (1)$$

Figure 2 presents the inner-scaled mean velocity profiles for a smooth wall and a range of rough walls.$^7$ These results were obtained in zero pressure gradient turbulent boundary layer flow. The roughness function is observed to displace the mean profiles for the rough surfaces below that on the smooth wall. Of particular note is the similarity in the mean profile shape that is observed for $y^+ > 100$ among the smooth- and rough-wall profiles. Similarity (or lack thereof) in the mean flow can be most readily examined when the profiles are plotted in velocity-defect form ($U^+ - U^+$). This will be illustrated later in the paper. The invariance observed in the mean velocity profiles in the overlap and outer part of the boundary layer implies that the direct influence of the roughness is confined to the inner layer. This notion is termed wall similarity.$^{11}$ The concept of wall similarity is not simply a curiosity for turbulence theoreticians. It has numerous practical implications. First, the utility of the roughness function itself hinges on similarity in the mean flow. If roughness affects the fundamental shape of the mean profile, the amount the profile is shifted from the smooth-wall case loses its physical relevance. At present most engineering tools for the prediction and modeling of rough-wall flows rely on wall similarity. Computational fluid dynamics codes routinely utilize the roughness function concept to account for the effect of surface roughness. Turbulence modeling with wall functions does so explicitly. This treatment typically requires the user to specify $k_s$ for the surface and the roughness function for Nikuradse's sand$^1$ is invoked to predict the velocity profile and wall shear stress for the rough wall. Other turbulence models including rough-wall $k-\varepsilon$ models$^{12}$ also
employ the roughness function to simulate flow over roughness. Furthermore, analytical approaches that allow scale up from laboratory-scale to engineering-scale also assume wall similarity.13,14

Townsend’s13 Reynolds number similarity hypothesis, with subsequent extensions by Perry and Chong16 and Raupach et al.,11 more formally expresses the concept of wall similarity. Townsend’s hypothesis states that at sufficiently high Reynolds number, the turbulent motions outside the roughness sublayer are independent of the wall boundary condition except in the role it plays in modifying the outer velocity ($U_{*}$) and length scales ($\delta$). The underlying assumption of Townsend’s hypothesis is that there is significant separation of scales between the boundary layer thickness ($\delta$) and the roughness height ($k$). It should also be noted that the roughness sublayer is generally thought to extend a few roughness heights from the wall.11 However, this simple definition of the roughness sublayer does not apply to all roughness types.7

This paper focuses on the authors’ research in rough-wall-bounded flows that has been carried out over the past decade and a half. The goal of this work is to critically evaluate the concept of wall similarity between rough- and smooth-wall flows and then, if valid, utilize the concept of similarity to determine the roughness scales that best predict frictional drag. The paper relies heavily on experiments that the authors have performed over a range of roughness types. The mean flow, Reynolds stresses, higher order statistics, turbulence spectra, and spatial correlations are all compared between rough- and smooth-wall flows. Additionally, the roughness scales that predict the onset of roughness effects, the shape and extent of the transitionally rough regime and fully rough behavior are also presented. While additional results from other research groups are included for comparison, this is not meant to be a thorough review of all the excellent work that has been accomplished in this field. A notable collection of recent rough wall studies can be found in Nickels.17

II. EXPERIMENTAL FACILITIES

The work presented has been obtained using facilities in the Hydromechanics Laboratory at the United States Naval Academy. The boundary layer experiments were conducted in two re-circulating water tunnels. All of the measurements were made in zero pressure gradient flows. The test section of the larger facility is 0.4 m by 0.4 m in cross-section and is 1.8 m in length, with a tunnel velocity range of 0–8.0 m/s. The test section of the smaller facility is 2 m long, 0.2 m wide and nominally 0.1 m tall, with a freestream velocity range of 0–1.5 m/s. Velocity measurements were made using a TSI FSA3500 two-component, fiber-optic laser Doppler velocimeter (LDV). Planar measurements were also obtained with a TSI particle image velocimetry (PIV) system. Details of experiments in the smaller facility can be found in Volino et al.,18 while details of the experiments in the larger facility can be found in Schultz and Flack5 and Flack et al.7

Skin-friction measurements were conducted in two turbulent channel flow facilities. The skin friction was obtained in the fully developed region of the flow via the flow rate and the streamwise pressure gradient. The smaller fully developed channel flow facility has a height ($H$) of 10 mm, a width ($W$) of 80 mm, and a ($L$) length of 1.6 m yielding an aspect ratio ($W/H$) of 8. The flow channel sits in and draws water from a large, quiescent basin. The bulk mean velocity in the channel ranges from 0.6 to 6.3 m/s yielding a Reynolds number ($Re_H$) range of 5800–64000. Seven taps pressure taps in the fully developed region $\sim$60 H–140 H from the inlet trips were used to measure the streamwise pressure gradient in the channel. Further details of the channel flow facility can be found in Flack et al.8

The high Reynolds number turbulent channel flow facility test section is a channel 25 mm in height ($H$), 200 mm in width ($W$), and 3.1 m in length ($L$), yielding an aspect ratio ($W/H$) of 8. Two pumps operating in parallel generate a bulk mean velocity of 0.4–11.0 m/s in the test section, resulting in a Reynolds number based on the channel height and bulk mean velocity ($Re_m$) ranging from 10 000 to 300 000. Pressure taps located $\sim$90H–110H downstream of the inlet trip are used to measure the streamwise pressure gradient in the channel. Further details of the channel flow facility can be found in Schultz and Flack.19

The towed plate experiments were conducted in the 115-m long towing tank facility. The width and depth of the tank are 7.9 m and 4.9 m, respectively. The towing carriage has a velocity range of 0–10.0 m/s. The towing velocity in the experiments was varied between 2.0 m/s and 3.8 m/s resulting

---

This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 136.160.90.132 On: Mon, 29 Sep 2014 13:50:02
TABLE I. Rough surfaces and flow conditions.

<table>
<thead>
<tr>
<th>#</th>
<th>Surface (number tested)</th>
<th>$k$ ($\mu$m)</th>
<th>$\delta/k$</th>
<th>$k^+$</th>
<th>$k^+$</th>
<th>$Re_{\theta \max}$</th>
<th>LDV</th>
<th>PIV</th>
<th>Tow Tank</th>
<th>$\Delta P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sprayed paint - orange peel</td>
<td>76</td>
<td>400</td>
<td>10.5</td>
<td></td>
<td>11900</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Sprayed paint sanded 120–60 grit</td>
<td>26–36</td>
<td>770</td>
<td>0.15–5.0</td>
<td></td>
<td>11400</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Woven mesh (4) 7, 22, 29, 34</td>
<td>320–2440</td>
<td>100–19</td>
<td>28–380</td>
<td>56–1150</td>
<td>12500</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Honed (diamond scratch)</td>
<td>26.3</td>
<td>1186–1000</td>
<td>2.3–26</td>
<td></td>
<td>27080</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Packed spheres w/o w/grit</td>
<td>305–610</td>
<td>88–51</td>
<td>12.5–211</td>
<td></td>
<td>28030</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Bars (2) 24, 34</td>
<td>1700</td>
<td>28</td>
<td>66</td>
<td>255</td>
<td>5000</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Bars cut into cubes</td>
<td>1700</td>
<td>28</td>
<td>66</td>
<td>255</td>
<td>5000</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A wide range of roughness has been tested in these facilities, as listed in Table I. These include two-dimensional roughness (bars), three-dimensional regular roughness (mesh, packed spheres, pyramids and cubes), and three-dimensional irregular roughness (paints, sandpaper, packed spheres with grit, honed, and grit blasted). The size of the roughness has also been varied significantly with $1186 < \delta/k < 16$, spanning the hydraulically smooth, transitionally rough, and fully rough regimes.

III. SIMILARITY BETWEEN SMOOTH- AND ROUGH-WALL FLOWS

As stated in the Introduction, similarity between smooth- and rough-wall boundary layer flows is very important since most predictive engineering tools rely on it. It is well known that surface roughness typically increases the wall shear stress and the boundary layer thickness and, depending on the size of the roughness, disrupts the near-wall coherent structures. However, an important question is whether modifications due to roughness are confined to the roughness sublayer or do the complex interactions between the coherent structures in the boundary layer influence the outer layer. The discussion of similarity in the mean flow and turbulence statistics presented here will focus on four experiments, incorporating many of the roughness types listed in Table I.

A. Behavior of small roughness over a large Reynolds number range [Surface # 6]

Wall similarity for rough surfaces cannot generally be expected to hold if it is violated for the limiting case where the Reynolds number is sufficiently high and the roughness is small compared to the boundary layer thickness. That is, the case where significant scale separation exists between the viscous length ($l_v$) and the roughness height, and between the roughness height and the boundary layer thickness ($l_v < k < \delta$). The first experiment discussed addresses this limiting case. This work is unique in that it covers a wide Reynolds number range, spanning the hydraulically smooth to the fully rough flow regime for a single rough surface, while maintaining a roughness height that is a very small fraction of the boundary layer thickness. Two test plates were used in this study; one smooth and one rough surface, geometrically similar to the honed pipe roughness tested in the Princeton Superpipe facility (Ref. 4). Figure 3 shows the mean flow in velocity defect form along with the Reynolds stresses. The velocity defect is normalized by the friction velocity while the Reynolds stresses are normalized by the wall shear stress. Excellent collapse is noted in the velocity...
defect profiles (Figure 3(a)) indicating similarity for the mean flow in the outer layer. The Reynolds stresses (Figures 3(b)–3(d)) also show agreement well within experimental uncertainty in the outer layer. These results support Townsend’s Reynolds number similarity hypothesis that the flow is insensitive to the wall boundary condition except for the role it plays in setting the outer length and velocity scales provided the Reynolds number is high and scale separation is large. Although not presented here, similarity in the outer layer is also observed in the quadrant-decomposed Reynolds shear stress and in the turbulent transport of the Reynolds stresses (Ref. 5). Further details of this work, as well as additional turbulence results, can be found in Schultz and Flack. 5 It is interesting to note the recent study of Chung et al. 25 that found, through direct numerical simulations, that the outer flow is highly insensitive to the imposed wall boundary condition. Specifically, comparisons were made between simulations with the typical no-slip wall boundary condition and with a shear stress wall boundary condition at the same friction Reynolds number. Differences in the mean flow, Reynolds stresses, and higher order turbulence statistics were isolated to a region close to the wall with collapse of these quantities in the outer flow. These results seem to indicate that the wall simply serves to set the boundary condition for the outer flow through the wall shear stress while the details of how the stress itself is generated is of little consequence, as originally postulated by Townsend.

B. Boundary layer similarity for large relative roughness [Surfaces #3 and 4]

While wall similarity is observed for the limiting case where the roughness height is a very small fraction of the boundary layer thickness, it is of interest to examine cases in which the relative roughness is not small. Therefore, the objective of the second experiment was to investigate the effect of increasing relative roughness height on the outer layer turbulence statistics. Jimenez 26 proposed in his review paper on rough-wall flows that similarity was expected to hold for $\delta/\kappa > 40$. The range of roughness used in this study spans from where similarity is expected to hold ($\delta/\kappa = 110$)
FIG. 4. Mesh and sandgrain roughness $k/\delta < 0.1$: (a) Velocity defect, (b) streamwise Reynolds normal stress profiles, (c) wall-normal Reynolds normal stress profiles, and (d) Reynolds shear stress profiles. Therefore, the limiting relative roughness height where similarity holds is explored. Seven surfaces were tested in this study. One was a smooth cast acrylic surface and the other six were rough surfaces. Three surfaces were covered with 80, 24, and 12 grit wet/dry sandpaper. The remaining three were covered with woven wire mesh with pitch to diameter ratios of 6.25 for the fine mesh, 4.58 for medium mesh, and 8.45 for coarse mesh. The mean velocity profile in defect form is shown in Figure 4(a). Excellent agreement between smooth- and rough-wall profiles is observed indicating similarity in the mean flow. Mean flow similarity for large roughness is further supported by the work of Castro who demonstrated similarity for $\delta/k > 5$. However, it was recently noted by Castro et al. that the velocity defect profile can hide variations in the wake strength. They therefore advocate the use of a diagnostic plot of $u'/U$ vs. $U/U_e$ to compare rough and smooth wall boundary layers. The Reynolds stress profiles (Figures 4(b)–4(d)) also indicate similarity in the outer layer, even in flows where $k$ is a significant fraction of $\delta$. Based on these results, comparatively large roughness elements that have a significant effect on the mean flow in the inner layer can be considered a small perturbation to the boundary layer with regards to the outer layer. Additionally, there does not appear to be a critical relative roughness height above which modifications to the turbulence are observed throughout all or most of the boundary layer. Instead, roughness effects are observed in the outer layer only as the roughness sublayer, a region extending a few roughness heights (or equivalent sand roughness heights) above the roughness, begins to extend into it. Further details of the mean flow results can be found in Connelly et al. while additional details of the turbulence, including higher order statistics, and quadrant decomposition, is found in Flack et al.

C. Turbulence structure for smooth- and rough-wall boundary layers [Surface # 4]

The experiments that have been described thus far have largely examined the concept of similarity in the context of the normalized mean flow and turbulence statistics. The third experiment focused on the turbulence structure over smooth- and rough-walls, as documented through spectra.
of the fluctuating velocity components and two-point spatial correlations. Both LDV and PIV measurements were obtained for flow over a woven wire mesh, with mesh spacing of $t = 1.69$ mm, and mesh wire diameter of 0.26 mm. The resulting peak-to-trough roughness height is $k = 0.52$ mm. Measurements were made in the streamwise-wall normal plane, and in two streamwise-spanwise planes located at $y/\delta = 0.1$ and 0.4. These correspond to $y/k_s = 2.0$ and 7.9, respectively, locations within and outside of the roughness sublayer. Premultiplied spectra of $u_w$, $v_v$, and $-u_wv_v$ are shown in Figure 5. At $y/\delta = 0.1$, there is a difference between the rough- and smooth-wall $u_u$ spectra at low wavenumbers. At higher wavenumbers for $u_u$ and for all wavenumbers in $v_v$ and $u_wv_v$, the smooth- and rough-wall spectra are similar. The overall difference in $u_u$ between the two cases is about 20%, which is large enough to indicate a possible difference between the cases, but still within the combined uncertainty of the measurements. At $y/\delta = 0.4$, the rough and smooth cases agree to within 3%. The rough-wall boundary layer contains significantly more turbulent energy than the smooth wall, but when scaled with $U_\tau$, the similarity between the rough-and smooth-cases is clear in the outer flow. Vector plots from PIV images (not shown here) indicate that hairpin packets are a prominent feature of the rough-wall boundary layer, much the same as its smooth-wall counterpart. The packets have a characteristic inclination angle and size which scales on the boundary layer thickness, and these quantities are consistent between the rough- and smooth-wall cases. This is supported further with the spatial correlations. A sample two-point correlation of the streamwise fluctuating velocity $R_{uu}$ centered at $y_{ref}/\delta = 0.4$, is shown in Figures 6(a) and 6(b). Figure 6(c) shows streamwise slices through the correlations, passing through the self-correlation peaks, while Figure 6(d) shows wall-normal slices passing through the self-correlation peaks. The rough- and smooth-wall results appear similar except in the near-wall region. These results indicate a great deal of structural similarity between rough- and smooth-wall flows. Further details of this work including additional results and discussion of turbulence structure can be found in Volino et al.\textsuperscript{18}

D. Boundary layer similarity for two-dimensional and periodic roughness

[Surfaces # 8 and 9]

The experiments that have been discussed thus far indicate similarity of the mean flow, Reynolds stresses, and turbulence structure in the outer layer for three-dimensional roughness. Previous experiments by other researchers (i.e., Refs. 30–32) have indicated that two-dimensional roughness causes a much larger disturbance to the boundary layer which can generate significant differences in the outer layer turbulence statistics and structure. The observation that two-dimensional roughness may create a stronger perturbation than three-dimensional roughness stands to reason. Since two-dimensional roughness spans the entire width of the flow field, fluid parcels traveling near the wall are forced to move vertically over the roughness element as no path exists to go around the roughness in the spanwise direction. The next set of experiments investigates two-dimensional roughness.
Two sets of transverse square bars were tested in the smaller boundary layer facility. The bar heights were \( k = 0.23 \) mm and 1.7 mm. The bars were spaced with a streamwise pitch (\( p \)) of 8\( k \). This pitch was shown by Furuya et al.\(^{33}\) to create the maximum roughness function for a given roughness height. Flow visualizations\(^{33}\) indicate that with this spacing, the flow is able to completely reattach behind a roughness element before it encounters and soon separates from the next element. As such, it can essentially be thought of as repeated tripping of the boundary layer. Tests were also performed on surface with rows of staggered cubes, with a cube height and spanwise spacing between cubes of 1.7 mm and streamwise pitch between rows of \( p/k = 8 \). Schematics of the small bars and larger cubes are shown in Figure 7. The cube surface was tested to investigate whether the streamwise periodicity of the roughness or the two-dimensionality of the roughness itself was the major contributor to the strong perturbation that is observed.

As demonstrated for the previous roughness, mean flow similarity is observed for all three rough surfaces (Fig. 8(a)). This is remarkable considering the fact that the large bars produce a very large disturbance to the flow (\( \delta/k_s = 2.5 \)). Considering the Reynolds stresses (Figs. 8(b)–8(d)), the transverse bars show differences in the outer layer, especially in the wall-normal and Reynolds shear
stress. This confirms results obtained by Krogstad and Antonia for transverse rods with a $p/k = 4$ at $\delta^+ = 2050$ a similar friction Reynolds number as the present large bar case, $\delta^+ = 1790$. The Reynolds stresses for the three-dimensional cubes are nearly similar to the smooth-wall results.

This raises a fundamental question for rough-wall boundary layer flows. Is Townsend’s Reynolds number similarity not valid for two-dimensional roughness? In order to further answer this question, Efros and Krogstad tested a transverse bar roughness at a significantly higher Reynolds number, addressing one of the fundamental assumptions of Townsend’s hypothesis. Results of their study are shown in Fig. 9. Good agreement is observed in both the mean flow and turbulence statistics between the smooth- and rough-wall results. It appears that that the differences between smooth-and two-dimensional rough walls diminish at high Reynolds number and higher $\delta/k$. Thus, similarity likely holds for two-dimensional roughness. However, the conditions that the Reynolds number is sufficiently high and the roughness is small compared to the boundary layer thickness must be more strictly met than for three-dimensional roughness.

All of the present author’s experiments for three-dimensional roughness support the notion of boundary layer similarity between rough- and smooth-wall flows, as evidenced by mean flow, Reynolds stresses, higher order statistics, turbulence spectra, and spatial correlations. Similarity also holds for two-dimensional roughness at sufficiently high Reynolds number and $\delta/k$. These results indicate that the outer layer is largely independent of surface condition except for the role that the wall conditions have on setting the length ($\delta$) and velocity ($U_{\tau}$) boundary conditions for the outer flow in accordance with Townsend’s Reynolds number similarity hypothesis.

IV. PREDICTION OF FRICTIONAL DRAG

The next area of focus is to utilize the similarity observed in the outer layer to develop engineering correlations for the prediction of frictional drag. Our philosophy has been to limit the parameters in
FIG. 9. Bar roughness: (a) Velocity defect, (b) streamwise Reynolds normal stress profiles, (c) wall-normal Reynolds normal stress profiles, and (d) Reynolds shear stress profiles.

the correlation to information that can be obtained solely from the surface topography. Thus we are excluding any information that requires hydrodynamic testing.

Musker\textsuperscript{36} introduced the concept of basing the predictive roughness scales on moments of the probability density function (pdf) of the surface topography. These moments yield statistical quantities such as the root-mean-square roughness height ($k_{rms}$), the skewness ($s_k$) of the pdf, and the kurtosis ($k_u$) of the pdf. The skewness is positive if a surface has more peaks (i.e., surface deposits) and negative if the surface has more valleys (i.e., corrosion, pitting). The kurtosis is an indicator of the range of scales in the roughness. A Gaussian surface has $s_k = 0$ and $k_u = 3$.

Other surface information that may be important to frictional drag is the mean absolute surface elevation ($k_a$), the peak-to-trough roughness height ($k_t$), the effective slope, the spectra, and a density parameter. $k_t$ is a measure of the largest surface features and is determined as the difference in elevation between the highest peak and the lowest trough in a given sampling length averaged over several samples. The effective slope ($ES$) defines the steepness of the surface features and has been used to classify “wavy” surfaces for $ES < 0.35$ (Refs. 37 and 38) where the roughness function no longer scales on the roughness height. The definition of the $ES$ is given in Eq. (2) where $L_s$ is the sampling length, $r$ the roughness amplitude, and $x$ the streamwise direction:

$$ES = \frac{1}{L_s} \int \left| \frac{\partial r}{\partial x} \right| \, dx.$$  (2)

Predictive scales based on surface statistics will likely need to be supplemented by a density parameter for sparse roughness. Solidity is one commonly used parameter to account for roughness density. Jimenez\textsuperscript{26} collated frictional drag results from a range of tests based on the solidity parameter ($\lambda$) of Schlichting,\textsuperscript{39} defined as the total projected frontal roughness area per unit wall-parallel projected area. A clear delineation in the frictional drag occurred at $\lambda \approx 0.15$. For $\lambda < 0.15$, the roughness elements are sparse and the frictional drag increases with increasing roughness solidity. For these conditions, the roughness elements individually contribute to the skin friction. For
non-sparse surfaces ($\lambda > 0.15$), the roughness elements have a shielding effect causing the frictional drag to decrease with increased solidity. Alternate versions of the solidity parameter have been proposed by a number of researchers, as reviewed by Flack and Schultz.\textsuperscript{45}

Unless the surface roughness is very homogenous, all of these measures are dependent on the size of the sampling region and filtering of the roughness height information.\textsuperscript{46} Ideally, a predictive correlation should include a prescribed sample size and filter range based on a defined roughness scale. The spectra of the surface elevations may be a useful tool to determine the filter range.

A key step to determining the surface scales that are responsible for the roughness-induced momentum deficit is to map the roughness function, $\Delta U^+$, over a range of roughness Reynolds number, $k^+ = ku_\tau/\nu$. These maps can be experimentally obtained in a range of ways. These include boundary layer velocity profiles, pressure drop measurements in fully developed internal flows, towing tank tests on flat plates, or the measurement of the frictional drag on rotating disks or cylinders as described by Schultz and Myers.\textsuperscript{47} A typical roughness function map is shown in Figure 10(a), where three flow regimes are observed. When $k^+$ is small, the perturbations generated by the roughness elements are completely damped out by the fluid viscosity. For this condition, the flow is hydraulically smooth and $\Delta U^+ = 0$. As $k^+$ increases, viscosity no longer damps out the eddies created by the roughness elements and form drag on the elements, as well as the viscous drag, contributes to the overall skin friction. This is the transitionally rough regime. As $k^+$ increases further, the skin friction is independent of Reynolds number, and form drag on the roughness elements is the dominant mechanism. In this fully rough regime, the roughness function reaches a linear asymptote. This corresponds to the skin-friction coefficient becoming independent of Reynolds number. The important features of this plot and related outstanding issues are: (i) the roughness Reynolds number where the surface ceases to be hydraulically smooth, (ii) the shape of the roughness function in the transitionally rough regime, and (iii) the roughness Reynolds number where the surface starts to exhibit fully rough behavior.

An important part of answering these questions is determining which measure of the roughness length scale (used to obtain the roughness Reynolds number) yields the best collapse of the roughness function data from disparate roughness types. It is worth noting that a different scale or combination of scales is likely required in each regime. Our approach has been to tackle each regime separately, as described below. Additionally, the correlations are developed for non-sparse, irregular roughness, indicative of many roughness types occurring in engineering applications.

### A. Fully rough regime

Collapse of roughness functions in the fully rough regime is achieved if the equivalent sand grain roughness height ($k_s$) is used as the scaling parameter, as demonstrated in Figure 10(b). However, as discussed previously, $k_s$ itself is not a physical scale and can only be determined from

![Figure 10. Roughness function for a range of roughness types using (a) $k = k_t$, (b) $k = k_s$. Note plot (b) only includes surfaces for which data in the fully rough regime have been obtained since this is necessary to specify $k_s$.](image)
hydrodynamic tests of the surface. Thus in order to provide an engineering predictive tool, \( k_s \) needs to be related to actual physical roughness scales. Flack and Schultz\(^4\) determined that the root-mean-square roughness height (\( k_{rms} \)) and the skewness (\( s_k \)) of the roughness surface elevation pdf had the strongest correlation with \( k_s \) yielding a function that best collapses a wide range of roughness results. Graphical representation is shown in Figure 11 and further discussion of the correlation is included in Flack and Schultz.\(^{45}\) While the constants in Eq. (3) provide the best fit of the range of rough surfaces tested, additional results would be useful to refine the correlation. Also note that Eq. (3) is limited to surfaces that are not strongly negatively skewed, as the correlation is limited to surfaces with \( s_k \geq -1 \):

\[
    k_s = f(k_{rms}, s_k) \approx 4.43 k_{rms} (1 + s_k)^{1.37}. \tag{3}
\]

If outer layer similarity in the mean flow for smooth and rough walls is valid, then roughness function results from laboratory scale can be used to predict frictional drag at full scale. Incorporating the analysis of Granville,\(^{14,48}\) Schultz,\(^{49}\) and Flack and Schultz\(^{45}\) describe a method to determine the overall frictional resistance coefficient, \( C_F \), for rough-wall boundary layer flow over a flat plate of length \( L \) from the roughness function \( \Delta U^+ \). If the roughness scale is the equivalent sandgrain roughness height \( k_s \), then the analysis is valid for all rough surfaces in the fully rough regime. The relationship between the frictional resistance and the non-dimensional roughness height (\( k_s/L \)) in the fully rough regime is given by Eq. (4) and graphically represented in Figure 12:

\[
    \sqrt{2} C_F = -2.186 \ln \left( \frac{k_s}{L} \right) + 0.495. \tag{4}
\]

Using Eq. (4) for the frictional resistance coefficient along with Eq. (3) for \( k_s \) allows for an engineering prediction based solely on measurements of surface topography. A key assumption used in Eq. (4) is that the fully rough regime is reached at \( k_s^+ = 70 \). New constants are required if this condition occurs at significantly different \( k_s^+ \). Studies have observed that fully rough conditions are reached at a range of \( k_s^+ \) and this likely depends on roughness type.\(^4\)\(^,5\)\(^,28\)\(^,50\)

Despite the apparent similarities between Figure 12 for external flows and the Moody diagram (Figure 1) for fully developed internal flows, there is an important difference between these two flow classes that is worth noting. For external flows, the fact that \( C_F \) is constant with Reynolds number for the fully rough condition should not be taken to imply that the skin-friction is invariant in the streamwise direction. To the contrary, the boundary layer is not in a self-preserving state as is the case with its internal flow counterpart. For a \( k \)-type roughness to be in an exactly self-preserving state (as defined by Rotta\(^{51}\)), the roughness height would need to gradually increase along the length of the plate. As demonstrated in Figure 12, for a given \( k_s/L \), there is a threshold \( \Re_L \) above which
$C_F$ is constant. For a given $L$, if $U$ increases, $Re_L$ increases but $C_F$ remains constant. For a given $U$, if $L$ increases, $Re_L$ increases but $C_F$ decreases as you move to a lower line of $k_s/L$. Therefore, there is spatial evolution of the skin-friction in the case of a fully rough boundary layer with a constant roughness height, similar to the smooth wall case. Neither is in a self-preserving state. This stands in contrast to fully developed internal flow (Ref. 3) which has a fixed outer scale.

### B. Onset of the transitionally rough regime

The next study described investigated the conditions when the surface cease to be hydraulically smooth and begins to show the effects of the roughness. This is the start of the transitionally rough regime. This set of experiments was performed in the small fully developed channel flow facility. Three sides of the channel are permanent, and the fourth side is a removable test surface. Experiments were first carried out using a cast acrylic surface as the fourth side which served as the smooth baseline test case. Sandgrain, ship bottom paint, and painted surfaces smoothed by sanding with progressively finer sandpaper (60–400 grit) were then tested. Measurements of the bulk flow rate and the streamwise pressure gradient were made at approximately 30 Reynolds numbers spanning the entire range of the facility. The wall shear stress was calculated from the measured pressure gradient in the channel. The skin-friction coefficient for the rough surface on one channel wall was determined from the overall shear stress and the measured skin-friction coefficient for the smooth wall as shown in Eq. (5):

$$c_{f_{\text{overall}}} = \frac{\tau_w}{\frac{1}{2} \rho U'^2} = \frac{c_{fS} + c_{fR}}{2}. \quad (5)$$

Roughness function results are shown on Figures 13(a) and 13(b) using $k_{rms}$ and $k_t$ as the scaling parameter, respectively. The sandpaper, with a large $k_{rms}$ and $k_t$ demonstrated the influence of roughness at $k_s^+ = 5$ and matched the behavior of uniform sandgrain roughness. The ship paints which have milder roughness deviated at roughness Reynolds numbers $k_{rms}^+ \sim 0.5$–0.7 or $k_t^+ \sim 10$. The painted surfaces smoothed by sanding showed less frictional resistance with progressively finer sandpaper. The roughness function for this type of surface roughness showed excellent collapse using the peak to trough roughness height, with the onset of roughness effects occurring for $k_t^+ = 9$. The skin-friction results for the surface sanded with 400-grit sandpaper were indistinguishable from the smooth wall results.

Based on these results, it appears that the largest surface features, represented by the peak-to-trough roughness height, are more important than an average roughness scale, represented by the rms roughness height, in predicting the onset of roughness effects. For a given roughness type, $k_t$ collapses the roughness function near the onset of the transitionally rough regime. However, $k_t$ was unsuccessful
FIG. 13. Onset of roughness effects scaled using (a) \( k = k_{rms} \) (b) \( k = k_s \). Note that the Nikuradse and Colebrook roughness functions are typically a function of \( k_s \). They are plotted here only to illustrate their shape. Their location on the abscissa is arbitrary.

in collapsing the roughness functions between these two classes of roughness. The peak-to-trough roughness height is a promising scaling parameter for the onset of roughness effects for a specific class of roughness. The ability to predict roughness effects becomes a more complicated problem when wider ranges of roughness surfaces are considered. This will likely require a combination of scaling parameters. Further details of these experiments including additional results can be found in Flack et al.\(^8\)

C. Transitionally rough regime

An important result exhibited in Figure 13 is that the roughness functions for the marine paint and painted-sanded surfaces do not exhibit either Nikuradse- or Colebrook-type behavior. As demonstrated for other surfaces, the shape of the roughness function in the transitionally rough regime is also roughness dependent. This leads to our next area of focus on the shape and extent of the roughness function in the transitionally rough regime. In order to fully investigate this regime, the roughness function needs to be mapped out over the entire range from hydraulically smooth to fully rough. This requires a wide range of roughness Reynolds number and is difficult to accomplish experimentally. This aspect of the study prompted the construction of the high Reynolds number turbulent channel flow facility. Baseline, smooth-wall results for this facility are reported in Schultz and Flack.\(^19\) Rough-wall experiments in the facility are currently underway. This research is focused on testing surface roughness which is systematically varied so that the surface scales that contribute to the roughness function behavior in the transitionally rough regime can be more easily identified.

V. CONCLUSIONS

While a number of important questions have yet to be answered, significant progress has been made in the understanding of flows over rough surfaces in recent years. This is largely due to a wealth of experimental studies over a range of roughness, the improving ability to compute these flows, and new insights into the problem.\(^53,54\)

Similarity between smooth- and rough-wall flows has been observed for a wide range of surface roughness. Roughness effects are confined to the inner layer, and in accordance with Townsend’s Reynolds number similarity hypothesis, the outer layer is insensitive to surface condition except in the role it plays in setting the length and velocity scales for the outer flow. This is even the case for two-dimensional roughness at sufficiently high Reynolds number and scale separation between the roughness and the boundary layer thickness.

The practical implication of wall similarity is the development of prediction tools based on the roughness function. The roughness scales that best correlate the roughness function in the fully developed region are the \( rms \) roughness height and the skewness of the roughness surface elevation...
pdf. The peak-to-trough roughness height is the scale that indicates when a surface will no longer behave as hydraulically smooth. Identifying a predictive scale in the transitionally rough regime has proven to be more difficult since the shape of the roughness function in this regime is not universal. Additional experiments and computations that systematically change surface parameters are needed to predict frictional drag in this regime.

ACKNOWLEDGMENTS

The authors would like to acknowledge the Office of Naval Research, program manager Ron Joslin, for the financial support of this research. We also appreciate the enlightening discussions over the years with fellow roughness researchers, with special thanks to Lex Smits, Per-Age Krogstad, and Ian Castro for sharing their data. Finally, we are very indebted to the tremendous support we receive from the technical staff in the Hydromechanics Laboratory and machine shop at the United States Naval Academy.