The onset of roughness effects in the transitionally rough regime

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ABSTRACT

Understanding the relationship between a surface's topography and its hydraulic resistance is an important, yet elusive, goal in fluids engineering. Particularly poorly understood are the flow conditions at which a given surface will begin to show the effects of roughness in the form of increased wall shear stress above that of the hydraulically smooth wall. This phenomenon is the focus of the present study. The results from a small scale fully-developed turbulent channel flow facility are presented for a hydraulically smooth wall and three types of rough surfaces (a sandpaper surface, two ship bottom paints and painted surfaces smoothed by sanding). Experiments were conducted over a Reynolds number (Re H) range of 5800-64,000 based on the channel height and the bulk mean velocity. The onset of roughness effects occurs for the sandpaper surface at k*0 = 1 or k* + = 5. This value of k* and the shape of the roughness function in the transitionally rough regime agree rather well with the results of Nikuradse (1933) for uniform sand. The frictional resistance of the two painted surfaces agree within experimental uncertainty despite a factor of two difference in k*0. The onset of roughness effects occurs at a maximum peak to rms roughness height, k', of ~10. Painted surfaces sanded with progressively finer sandpaper (60-220 grit) show roughness effects for k*0 = 0.5-0.7 or k* + = 9, while the 400 grit sandpaper displays hydraulically smooth conditions over the entire Reynolds number range. The roughness scale that best predicts the onset of roughness effects is k*, indicating that the largest roughness features have the most influence in determining when a surface ceases to be hydraulically smooth. It is also of note that the roughness functions for the marine paint and painted-sanded surfaces do not exhibit either Nikuradse or Colebrook-type behavior.

1. Introduction

Accurately predicting the increase in frictional drag due to surface roughness remains an important objective of fluids engineering research. Predictive models have been proposed for rough wall flows in the fully rough regime, where pressure drag on the roughness elements dominates. However, the frictional drag behavior in the transitionally rough flow regime, where both viscous and pressure drag are significant, is much more poorly understood. This is unfortunate since most engineering flows, including flows over ship hulls and turbine blades as well as fluid transport typically routinely operate in this regime. An important question in the transitional regime is how smooth is "hydraulically smooth". Understanding the roughness scales that produce the onset of roughness effects is important for determining manufacturing tolerances and polishing levels necessary to produce test models that remain free of roughness effects. This is especially critical for flows operating at high unit Reynolds numbers. Currently, the onset of roughness effects and amount of additional drag due to roughness in the transitionally rough regime is only reliably known for a few surfaces that have been studied in detail. This leaves the frictional drag behavior of most surfaces in the transitionally rough regime unclear.

The amount of frictional drag due to surface roughness is dependent on many surface parameters including roughness height (k), shape, and density. A common measure of the moment loss due to roughness is the roughness function (\( \Delta U^* \)) which is the downward shift in the mean velocity profile when expressed in inner variables. A plot of \( \Delta U^* \) as a function of the roughness Reynolds number, k* = k UD/v, for a wide range of surfaces (Flack and Schultz, 2010) indicates three distinct regimes (Fig. 1). When the roughness Reynolds number, k*, is small, the flow is hydraulically smooth (i.e. \( \Delta U^* = 0 \)). In this case the perturbations generated by the roughness elements are completely damped out by the fluid viscosity, creating no additional frictional drag. As k* increases, the flow begins to show the effects of the roughness and becomes transitionally rough. In the transitionally rough regime, viscosity is no longer able to damp out the turbulent eddies created by the roughness elements and form drag on the elements, as well as the viscous drag, contributes to the overall skin friction. As k* increases...
Fig. 1. Roughness function results for a range of surfaces scaled on $k_s$. Further, the roughness function reaches a linear asymptote. This asymptotic region at large values of $k^*$ is termed the fully rough regime. In this regime, the skin friction coefficient ($c_f$) is independent of Reynolds number, and form drag on the roughness elements is the dominant mechanism responsible for the momentum deficit in the boundary layer. The roughness functions for all surfaces will asymptote to a universal fully rough line if the equivalent sandgrain roughness height scaled using the viscous length scale, as shown in Fig. 2 (Flack and Schultz, 2010). The equivalent sandgrain roughness height is the roughness height that produces the same roughness function as the uniform sand roughness of Nikuradse (1933) in the fully rough regime. But, as indicated in Fig. 2, collapse in the fully rough regime does not ensure collapse in the transitionally rough regime.

The relationship between the roughness Reynolds number and the roughness function for a generic surface in the transitionally rough regime remains unknown. The values of $k^*$ (or $k_s^*$), the equivalent sandgrain roughness height, are used as the characteristic roughness length scale, as shown in Fig. 2 (Flack and Schultz, 2010). The equivalent sandgrain roughness height is the roughness height that produces the same roughness function as the uniform sand roughness function. Ligrani and Moffat (1986) found the transitionally rough regime to span $15 < k_s^* < 50$ for a close-packed sphere bed. This range is reported as $3.5 < k_s^* < 30$ for honed pipe roughness by Shockling et al. (2006) and is given by Schultz and Flack (2007) as $2.5 < k_s^* < 25$ for a similar surface created by surface scratches. Langelandsvik et al. (2008) indicate that the range of the transitionally rough regime is $1.4 < k_s^* < 18$ for commercial steel pipe. Musker and Lewkowicz (1978) found that the onset of the fully rough regime ranged from $k_m^* = 17$ to 40 for ship hull roughness. Additionally, the shape of the roughness function in the transitionally rough regime also varies depending on roughness type and has been shown to have a monotonically changing slope for some roughness types and an inflectional behavior for other surfaces.

The objective of the current research is to identify the roughness scales that determine when a surface ceases to be hydraulically smooth. The scales used will be based solely on details of the surface roughness instead of parameters dependent on detailed flow surveys. The goal is to provide a predictive engineering tool for the onset of roughness effects.

2. Experimental methods

In order to effectively determine the onset of roughness effects for a given surface, one must be able to detect small differences in the wall shear stress. To address this challenge, a fully-developed plane channel flow facility was constructed. A schematic of the test section is shown in Fig. 3. The channel has a height ($H$) of 10 mm, a width of 80 mm, and a length of 1.6 m. The inlet has a contraction with an area ratio of 8:1. The working fluid is water, and the bulk mean velocity ($U$) in the channel ranges from 0.6 to 6.3 m/s. The facility is powered by a 1.5 kW 3-phase AC centrifugal pump, and the fluid velocity is controlled via a variable frequency controller. The flow channel sits in and draws water from a large, quiescent basin (diameter = 7.3 m, water depth = 0.45 m). The inlet contraction and outlet ducting of the channel are located at opposite sides of the basin to minimize inflow disturbances. Tests were conducted without inflow conditioning since pressure drop measurements with and without a honeycomb flow straightener yielded similar results.

The Reynolds number ($Re_H$) range in the facility is 5800–64,000 (based on the channel height and the bulk mean velocity), providing a variation in the viscous length scale of 3.4–28 µm on a smooth wall. Trip bars, 0.75 mm in height and 1.5 mm in thickness, spanning the channel width, were placed immediately downstream of the contraction on the top and bottom walls of the channel. The trip bars provide 15% blockage as recommended by Durst et al. (1998) to ensure that the flow in the test section is fully-developed and turbulent at the lower Reynolds number range. The facility has seven static pressure taps in the fully-developed region of the flow. The diameter of the static pressure taps is 0.8 mm. The first tap is located 60H downstream of the trip bars, and the taps are spaced 15H apart in the streamwise direction. Durst et al. (1998) showed that 60H is a sufficient length to obtain a fully-developed turbulent channel in terms of the wall shear stress and mean flow at $Re_H > 3000$. In the present study, only the five most downstream taps (>90H from the trips) were used to make the static pressure measurements.
The static pressure measurements were made using GE Druck LPM9000 differential pressure transducers. Three transducers with ranges of 200 mm, 500 mm, and 1000 mm of water, respectively, were employed. The accuracy of the differential pressure transducers is ±0.1% of the range. A Yokagawa ADMAG AXF flowmeter was used to measure the bulk flow rate through the channel. This flowmeter has a range of 0–380 lpm, with an accuracy of ±0.2% of the reading.

Three sides of the channel are permanent, and the fourth side is a removable test surface. Experiments were first carried out using a cast acrylic surface which served as the smooth baseline test case. Measurements of the bulk flow rate and static pressure were made at approximately 30 Reynolds numbers spanning the entire range of the facility. The flow rate and pressure measurements were sampled at a rate of 50 Hz for 30 s for each Reynolds number. The entire smooth wall experiment was replicated six times. The wall shear stress was calculated from the measured pressure gradient in the channel, as shown in the following:

\[ \tau_w = -\frac{H \, dp}{dx} \]

The skin friction coefficient, \( C_f \), was calculated using the following relationship:

\[ C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} \]

The overall precision and bias uncertainty for the smooth wall \( C_f \) at 95% confidence is ±1.2%. The results for the smooth wall channel are presented in Fig. 4. Also shown for comparison are the experimental data of Monty (2005) as well as the power law correlations of Dean (1978) and Zanoun et al. (2003).

The present smooth-wall results fall between the correlations of Dean (1978) and Zanoun et al. (2003) over the entire Reynolds number range. For the lower Reynolds numbers, the present results agree well with both the experimental data of Monty (2005) and the correlation of Dean (1978). For example, at \( Re_w < 30,000 \), the present \( C_f \) results agree within 4% with Dean's correlation. At the higher Reynolds numbers range, the present results fall systematically below those of Monty and Dean until for \( Re_w > 60,000 \) they lie roughly between the correlations of Zanoun et al. and Dean. The disparity at higher Reynolds numbers with the results of Monty (2005) could be attributed to the differences in channel aspect ratio for the experiments (8:1 present case, 11.7:1 Monty).

In order to assess the applicability of the channel flow apparatus to determine the rough wall skin friction and roughness function, a baseline rough surface was also tested; 220-grit sandpaper. This surface was chosen because it is well defined, readily available, and has been previously tested by Schultz and Myers (2003) using a number of roughness function determination methods over a wide Reynolds number range. The experiments were carried out using the same procedure that was outlined for the smooth wall with the exception that the entire lower surface is covered with roughness. The rough-wall experiments were replicated four times. However, for the rough walls, the skin friction coefficient determined using Eqs. (1) and (2) represents the average skin friction of the smooth and rough wall as given by:

\[ C_f_{\text{exp}} = \frac{\tau_w}{\frac{1}{2} \rho U^2} = C_f_S + C_f_R \]

where \( C_f_S \) and \( C_f_R \) represent the skin friction coefficients on the smooth and rough wall, respectively. The skin friction coefficient for the smooth wall was determined using the present experimental results presented in Fig. 4. This allowed the rough-wall skin-friction coefficient to be determined using Eq. (3). There is a small contribution to the measured pressure drop from the shear stress on the side walls. However, this effect was studied by Monty (2005) who reported that no correction for the side walls is necessary when using Eq. (1) to obtain the wall shear stress along the centerline of the channel provided that the aspect ratio is 7 or larger. The contribution of the side wall shear stress has, therefore, been neglected in the present study. Based on Eqs. (1)–(3) and the results of the replicate experiments, the overall precision and bias uncertainty for the rough wall \( C_f \) at 95% confidence is estimated to be ±1.5%.

The approach taken here should be valid as long as the flow is not very far from hydraulically smooth conditions. As the roughness function increases for the rough wall and the flow approaches the fully rough regime, the channel will exhibit significant asymmetry, and the results for the smooth wall measured in a symmetric channel (Fig. 4) can no longer be expected to strictly hold. However, since this work focuses on identifying the point of departure of rough-wall flows from hydraulically smooth conditions, the present approach should be acceptable. It should be noted that the range of applicability of the approach taken here will be critically assessed when the roughness functions obtained using this methodology are compared with those previously found for the same surface. The skin friction coefficient results for the 220-grit roughness are presented in Fig. 5.

The \( C_f \) results for 220-grit sandpaper show that this surface displays significant roughness effects even at the lowest Reynolds number tested. Furthermore, the transitionally rough regime displays an inflectional dip that also characterizes the frictional resistance behavior of uniform sandgrain roughness (Nikuradse, 1933).
In order to determine the roughness function, $\Delta U^*$, for the rough surfaces, the similarity-law procedure of Granville (1987) for fully-developed internal flows was used. Granville's method states that the roughness function can be obtained as follows:

$$\Delta U^* = \sqrt[2]{R_e} - \sqrt[2]{R_e^2}$$ (4)

where $C_f$ and $C_k$ are evaluated at the same value of $R_e(c)^{1/2}$. Using error propagation techniques and the results of the replicate experiments, the overall precision and bias uncertainty for the rough wall $\Delta U^*$ at 95% confidence is estimated to be ±7% or ±0.15, whichever is larger. The roughness function results for the 220-grit sandpaper surface are presented in Fig. 6. The equivalent sand roughness height for the sandpaper surface was taken to be $k_t = 206 \mu m$ as obtained for the same roughness by Schultz and Myers (2003). As can be seen in Fig. 6, there is excellent agreement between the present results and those obtained by Schultz and Myers (2003) for $\Delta U^* < 5$. Both of these data sets agree very well with the roughness function for uniform sand found by Nikuradse (1933). For larger values of the roughness function ($\Delta U^* > 5$), the present results depart from both the results of Schultz and Myers (2003) and Nikuradse (1933) and are systematically higher. Furthermore, at large values of the roughness Reynolds number, the slope of the roughness function does not approach the expected asymptotic value of $k^{-1}$. This is most likely due to the asymmetric channel effects that were mentioned previously. The results presented in Fig. 6, therefore, provide some confidence in the use of the present technique to determine the roughness function for surfaces in which the roughness function is not too large. These results also point out, however, that the present methodology cannot be expected to provide accurate roughness function results in cases where the roughness function is large (i.e. $\Delta U^* > 5$).

In this paper, the behavior of two different classes of surfaces in the hydraulically smooth flow regime into the transitionally rough flow regime will be examined. These will include a range of painted surfaces that have been systematically smoothed by sanding and two types of ship bottom paints. The details of these surfaces along with the skin friction coefficients and roughness functions will be discussed in the following section.

### 3. Results and discussion

The surface statistics of peak to trough roughness height ($k_t$), root-mean-square roughness height ($k_{rms}$), the skewness ($Sk$) and flatness ($Ku$) of the roughness probability density function (pdf) are listed on Table 1. The surfaces were profiled with a Veeco Wyco NT9100 optical profilometer utilizing white light interferometry, with sub-micron vertical accuracy. The peak to trough roughness height is most dependent on the size of the sampling area since it identifies the largest roughness feature of the surface. A larger interrogation region would likely yield a larger $k_t$. Therefore the $k_t$ listed is the average over five sample regions. The surface statistics of $k_{rms}$ and $Sk$ were previously used by Flack and Schultz (2010) to predict the frictional drag on a rough surface in the fully rough regime. The skewness ($Sk$) is a quantitative way of describing whether the roughness has more peaks (positive) or valleys (negative). The kurtosis ($Ku$) is a measure of the range of scales.

Surface topographical maps and probability density functions (pdfs) for the copper and silicone based marine antifouling paints are shown in Figs. 7 and 8, respectively. Both surfaces display islands of peaks and troughs, with mild transition zones between these regions. As indicated in Table 1, the copper paint has a

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$k_t$ ((\mu m))</th>
<th>$k_{rms}$ ((\mu m))</th>
<th>Sk</th>
<th>Ku</th>
</tr>
</thead>
<tbody>
<tr>
<td>220-grit SP</td>
<td>305</td>
<td>36.9</td>
<td>0.10</td>
<td>2.84</td>
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<tr>
<td>Copper paint</td>
<td>152</td>
<td>20.8</td>
<td>-0.03</td>
<td>2.75</td>
</tr>
<tr>
<td>Silicone paint</td>
<td>135</td>
<td>10.0</td>
<td>-0.13</td>
<td>3.13</td>
</tr>
<tr>
<td>60-grit sanded</td>
<td>108</td>
<td>9.72</td>
<td>-0.49</td>
<td>3.71</td>
</tr>
<tr>
<td>80-grit sanded</td>
<td>94.0</td>
<td>4.91</td>
<td>-0.57</td>
<td>3.70</td>
</tr>
<tr>
<td>220-grit sanded</td>
<td>60.5</td>
<td>3.57</td>
<td>-0.39</td>
<td>3.42</td>
</tr>
<tr>
<td>400-grit sanded</td>
<td>46.3</td>
<td>1.55</td>
<td>-1.57</td>
<td>8.56</td>
</tr>
</tbody>
</table>

**Fig. 7.** Copper paint: (a) surface topographical map and (b) surface elevation probability density function.
slightly larger peak to trough roughness height \( (k_a) \) than the silicone paint for the interrogation region, however, the rms roughness height \( (k_{rms}) \) for the copper paint is approximately twice that of the silicone paint. This large difference in rms roughness height is not apparent in the measurements of skin friction coefficients, Fig. 9. The skin friction coefficient for the copper paint is slightly higher than that of the silicone paint, with the difference within the experimental uncertainty of the apparatus.

The roughness functions for the marine paints near the onset of roughness effects are shown in Figs. 10–12 for a range of scaling parameters. As expected from the skin friction results, \( \Delta U' \) does not scale solely on the rms roughness height, with the copper paint departing from hydraulically smooth condition at a higher \( K_{rms}^* \) than the silicone paint. Since the surface topography shows undulating regions of peaks and troughs, a slope parameter was also investigated. Eq. (5) shows the effective slope \( (ES) \), where \( L \) is the
sampling length, \( r \) is the roughness amplitude and \( x \) is the streamwise direction.

\[
ES = \frac{1}{L_1} \int_0^{L_1} |\nabla r| \, dx
\]  

(5)

Napoli et al., 2008 classified surfaces with effective slopes less than 0.35 as wavy. The effective slopes for the copper and silicone paints are 0.10 and 0.12, respectively. Including the slope parameter (Fig. 11) slightly improves the prediction of the departure from hydraulically smooth condition, however does not collapse \( \Delta U' \) for the range of roughness functions. This is expected since the effective slopes have very similar magnitudes. The peak to trough roughness height, \( k_r \), is used as a scaling parameter in Eq. 12. This parameter is able to predict the point of departure from hydraulically smooth fairly well and collapses the results for the entire range of roughness Reynolds numbers tested. This is a particularly useful parameter for marine paints since it is easily measured in practice. A drawback is that it is highly dependent on the size of the sampling domain. Based on these results for marine paints, roughness effects would be present for \( k_r > 10 \).

Using the boundary-layer similarity law scaling developed by Granville (1958, 1987), it is possible to scale these laboratory results to predict the impact of this hull paint roughness at ship scale. Further details of the method are offered in Schultz (2007). Considering a naval destroyer with a length of \( \sim 140 \) m, the onset of the transitionally rough regime (\( k_r \sim 10 \)) would occur at a ship speed of \( \sim 5 \) knots. By the time the ship reaches a cruising speed of 15 knots, the increase in frictional drag due to this roughness would be 2.3% (or 0.9% of the total drag). At 30 knots, the frictional drag penalty would be 8.0% (or 1.6% of the total drag). These results indicate that marine antifouling paints, as they are typically applied, can suffer significant frictional drag penalties. These penalties become much larger when the paint becomes fouled (Schultz, 2007; Schultz and Bendick, 2011).

The next series of roughness tested was painted surfaces sanded with progressively finer grade sandpaper, ranging from 60 to 400-grit. The painted surface was sanded repeatedly with alternating \( \pm 45^\circ \) orientations. The last pass of the sandpaper is apparent in the scratches observed in the surface topographical maps and pdfs for the various grades of sandpaper, shown in Figs. 13–16. As indicated on the surface maps, and the surface statistics (Table 1), the surface roughness has smaller amplitudes as the smoothing sandpaper becomes finer. With the exception of the 400-grit sanded surface which is nearly smooth, the surfaces are mildly negatively skewed indicating more troughs than peaks in the surface profile.

The influence of the sandpaper used for smoothing is also observed in the plot of the skin friction coefficient in Fig. 17. The skin friction deviates from the smooth curve at higher Reynolds number as the roughness scales decrease. The painted surface sanded with 400-grit paper is nearly indistinguishable from the smooth curve. This trend in departure from hydraulically smooth is also observed in the roughness function results, scaled using the rms roughness height (Fig. 18). The onset of roughness effects occurs for \( k_r' \) ranging from 0.5–0.7, indicating that a sanded surface could be classified as hydraulically smooth for \( \kappa_r' < 0.5 \). When scaled using

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig13}
\caption{60-grit sanded paint: (a) surface topographical map and (b) surface elevation probability density function.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig14}
\caption{80-grit sanded paint: (a) surface topographical map and (b) surface elevation probability density function.}
\end{figure}
the peak to trough roughness height (Fig. 19), $k^*_{V}$, the roughness functions collapse, with the onset of departure at $k^*_{V} = 9$. These results are valuable for model makers to determine when the surface can be classified as hydraulically smooth for the hydrodynamic conditions of the model test.

Consider, for example, a 1:36 scale model of a 140 m destroyer used for towing tank testing. If the top speed of the ship is 35 knots (18 m/s), then using Froude scaling, the top speed of the model is 3 m/s. Based on the ITTC friction line (Gillmer and Johnson, 1982) given below (Eq. (6)), the overall frictional drag coefficient is:

$$C_F = \frac{0.075}{(\log_{10}(Re_L) - 2)^2}$$

(6)

where $Re_L$ is the Reynolds number based on length. An estimate of the average friction velocity ($U_f$) can be obtained using the following equation:

$$U_f = U \sqrt{C_F \frac{k^*}{2}}$$

(7)

where $U$ is the towing speed of the model. In the present case, this yields a viscous length scale ($l_v = \sqrt{C_F}$) of 8.7 µm at the highest towing velocity. If one then uses $k^*_{V} < 9$ as the criteria for hydraulically smooth conditions, it is noted that the 60-grit and 80-grit sanded surfaces would both exceed this value and roughness effects would be expected. For the 220-grit sanded surface, $k^*_{V}$ would be ~7. Therefore, this surface finish would be acceptable in order to maintain hydraulically smooth conditions during the testing.

The roughness functions for all the surfaces tested are shown on Fig. 20 using $k^*_{V}$. The peak to trough roughness height yields collapse for the specific classes of roughness types, but does not collapse the roughness function for the entire range of surfaces. This highlights the need for additional parameters if collapse of the profiles is to be achieved. Admittedly, a universal scaling based on surface parameters may not exist, and the best solution may be to use different scaling parameters for various classifications of roughness types.
The shape of the roughness functions near the onset of roughness effects is less abrupt than the Nikuradse roughness function and has a steeper slope than the Colebrook roughness function.

Another important result highlighted in Fig. 20 is that the painted and sanded surfaces do not follow previously proposed roughness functions, including the Colebrook (1939) roughness function that is employed in the widely used Moody (1944) diagram. The shape of the roughness functions near the onset of roughness effects is less abrupt than the Nikuradse roughness function and has a steeper slope than the Colebrook roughness function.

4. Conclusions and future work

A facility has been constructed that can accurately detect the point of departure from hydraulically smooth conditions for a rough surface. The sandpaper, with a large $k_{rms}$ and $k_I$ demonstrated the influence of roughness at $k^+ = 5$ and matched the behavior of uniform sandgrain roughness (Nikuradse, 1933). The ship paints which have milder roughness deviated at roughness Reynolds numbers $k_{rms} \sim 0.5-0.7$ or $k^+ \sim 10$. The painted surfaces smoothed by sanding showed less frictional resistance with progressively finer sandpaper. The skin friction results for the surface sanded with 400-grit sandpaper were indistinguishable from the smooth wall results. The roughness function for this type of surface roughness showed excellent collapse using the peak to trough roughness height, with the onset of roughness effects occurring for $k^+ = 9$.

In the present study, the peak to trough roughness height collapsed the roughness function within two types of roughness classes tested. It was unsuccessful in collapsing the roughness function between these two classes of roughness. It appears that the largest surface features, represented by the peak to trough roughness height, are more important than an average roughness scale, represented by the $rms$ roughness height, in predicting the onset of roughness effects. Future experiments will test this hypothesis for a wider range of surfaces and map the roughness function throughout the entire transitionally roughness regime. Understanding the shape of the roughness function (monotonic, inflectional, etc.) within the transitional regime is important for predicting the effect of roughness over a wide Reynolds number range.

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