SUBMIT ANSWER SHEET ONLY TO: usnastem@usna.edu by MARCH 21, 2021

Math Is Everywhere!
Even in SeaPerch ROVs

ROVs (Remotely Operated Vehicles) are used to collect oceanographic data, to perform underwater repairs and attach cables, to search for lost ships and cargo, to locate underwater mines and much more. SeaPerch is an example of a simple ROV you can build yourself—it comes as a kit and is made from PVC pipe and other inexpensive, easily obtained materials. Students around the world are building and using their SeaPerch to collect data and learn about science and engineering.

1. To begin to build your SeaPerch, you must cut the five 12-inch pieces of PVC pipe to the correct sizes:

- Two pieces – **2 ½** inches long
- Two pieces – **4** inches long
- Two pieces – **4 ½** inches long
- Four pieces – **1 ½** inches long
- Four pieces – **5** inches long

   a. If your PVC lengths are placed into a bag, what is the probability that you will reach in and select a piece that is 5 inches long?
   b. What is the average pipe length? What is the median pipe length? What is the mode pipe length?
   c. Of the 60 inches in total pipe length in your kit, how much remains unused after you have cut and assembled your SeaPerch?

2. The payload net is attached to the bottom of the SeaPerch and used to hold the ballast. The hydrostatic pressure experienced by an object is 0 psi (pounds per square inch, lb/in^2) at the water's surface, and increases by 0.445 psi for every foot an object descends under the water.

   a. Calculate the hydrostatic pressure experienced by the payload if your test-tank is 5 ft deep.
   b. Calculate the area of the payload support in in^2 (use the diagram for reference dimensions).
   c. Pressure is defined as the ratio of force per unit area: $\boldsymbol{P = \frac{F}{A}}$. Now that you know the hydrostatic pressure and the area, calculate the hydrostatic force experienced by the bottom surface of your SeaPerch, in lbs (assume continuous surface).

3. You have souped up your SeaPerch with an alarm and a beacon light, both of which behave differently in water than they do in air.

   The speed of sound, $\boldsymbol{v}$, depends on the density, $\boldsymbol{\rho}$, of the medium and the Bulk modulus, $\boldsymbol{B}$, which is a measure of the medium's compressibility: $\boldsymbol{v} = \sqrt{\frac{\boldsymbol{B}}{\boldsymbol{\rho}}}$.

   a. Calculate the speed of sound in air ($B = 20.6\,psi,\ \rho = 1.22x10^{-7}\ lb\ s^2/in^4$)
   b. Calculate the speed of sound in H_2O($B = 3.12x10^5\ psi, \rho = 9.38x10^{-5}\ lb\ s^2/in^4$)

   Light bends, or refracts, as it crosses a boundary from one medium into another. Each medium has an index of refraction, $\boldsymbol{n}$, which is the ratio of the speed of light in a vacuum, $\boldsymbol{c}$, and the speed of light in the medium, $\boldsymbol{v}$: $\boldsymbol{n} = \frac{\boldsymbol{c}}{\boldsymbol{v}}$.

   c. If light travels at $c = 1.18x10^{10}\ in/s$ in a vacuum and at $v = 8.86x10^9 in/s$ in water, calculate the index of refraction for water.

4. Your SeaPerch teammates are active in so many STEM activities in school!

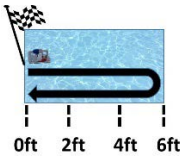
CLUES

1. **Ava, Liam Mia, Noah and Emma are each in two STEM Clubs.**
2. **All five are either Sea Perch or Robotics, but not both.**
3. **Each is also in a unique STEM Club, which the others are not in.**
4. **Liam and Mia have pet cats. The others have no pets.**
5. **The Chemistry Club Member and the Physics Club Member are the only two that are in the Robotics Club.**
6. **The Math Team member and the Chemistry Club Member have no pets, and volunteer for STEM Outreach together.**
7. **Ava and Mia are in a club together.**
8. **Noah, who is in the Robotics Club, and Emma, who is in the Sea Perch Club, do not know each other, but the member of the Cyber Team is trying to introduce them.**

Determine the two STEM clubs that each student has joined, using the given clues.

	AVA	LIAM	MIA	NOAH	EMMA
SEA PERCH					
ROBOTICS					
CHEMISTRY CLUB					
PHYSICS CLUB					
MATH TEAM					
FUTURE ENGINEERS					
CYBER TEAM					

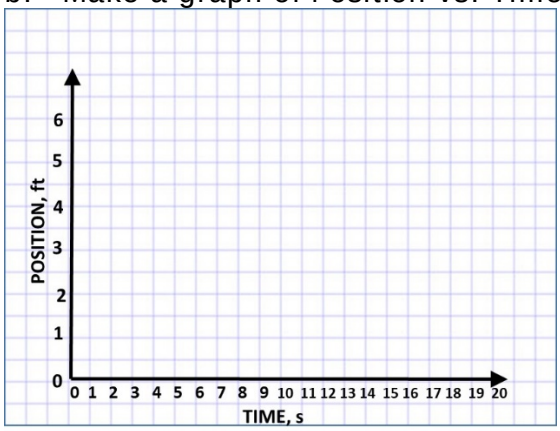
5. Below is data for a SeaPerch launch, back and forth across a 6-foot tow tank.



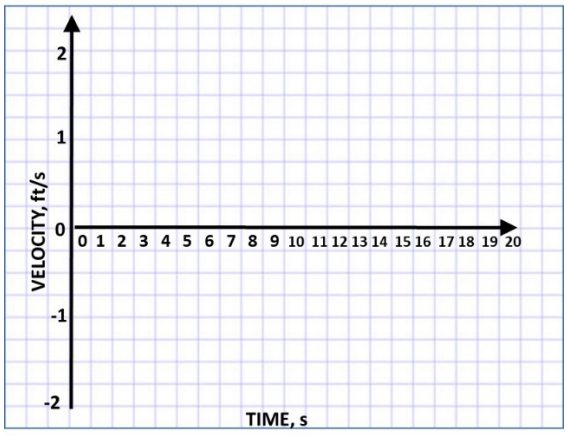
a. The formula for average velocity is $v = \frac{d_f - d_i}{t_f - t_i}$. Calculate the average velocity over each time interval and enter in table.

Time (s)	Position, d (ft)	Displacement (ft) $\Delta d = d_f - d_i$	Time elapsed (s) $\Delta t = t_f - t_i$	Velocity, v(ft/s)
0	0	----------	----------	0
5	2	$\Delta d = d_f - d_i$ $= 2 - 0 = 2ft$	$\Delta t = t_f - t_i$ $= 5 - 0 = 5s$	$v = \frac{d_f - d_i}{t_f - t_i} = \frac{2ft}{5s} = 0.4\ ft/s$
8	4	$\Delta d = d_f - d_i$ $= 4 - 2 = 2ft$	$\Delta t = t_f - t_i$ $= 8 - 5 = 3s$	
12	6			
14	6			
17	4			
19	2			
20	0			

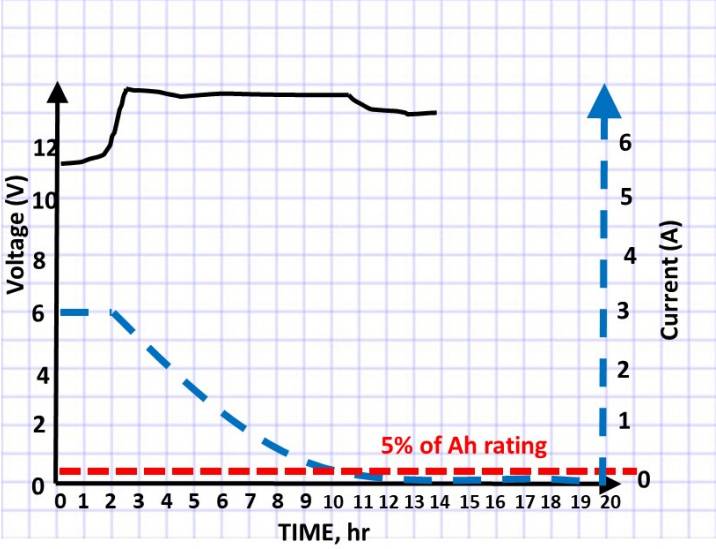
b. Make a graph of Position vs. Time



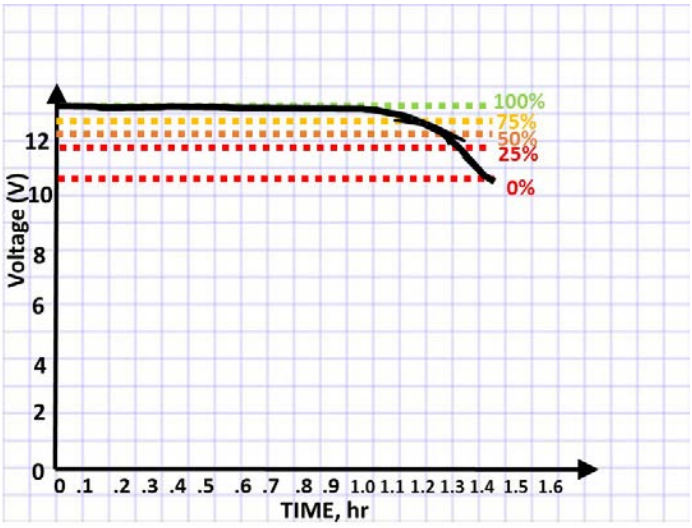
c. Make a graph of Velocity vs. Time



6. Once you have built your SeaPerch, you need to power it with a battery. The battery has a voltage of 12 volts and is rechargeable. The charging curve (black line) is shown here. You will see that the charging current (blue line) begins steady at 3A, and then drops off as the battery is charged. The battery is considered fully charged when the current is reduced to 5% of its rated amperage, marked with a red line on the graph.
    a. How long does it take to charge a battery?
    b. What is the final voltage on the charged battery?



7. The battery is designed so that its discharge voltage remains relatively flat and remains a viable energy source for as long as possible.
    a. How long does the battery remain at least 75% charged?
    b. How long does it take for the battery to discharge from 75% to 0%?
    c. What is the voltage of a 12V battery that has 0% charge?



8. You need to make your SeaPerch neutrally buoyant so it will not float or sink in the water but just hover below the water line. When an object is submerged in a fluid, there is an upward force on the object that is equal to the weight of the fluid displaced by the object. When the density of the object is less than that of the fluid, the net force will be upwards and the object will float. If the density is greater, the object will sink. There are 14.4 cubic inches in a cup and the density of water is 0.58 oz/in^3.

   You have filled a 5-gallon bucket to the very top with water. When you placed your SeaPerch in the bucket, 5.5 cups of water spilled over.

   a. What is the buoyant force on your SeaPerch, in oz?

You carefully pushed the top of the perch so that it was completely submerged, (which is called neutrally buoyant), spilling an additional 2.5 cups of water. You have ballasts available, each of which has a mass of 147 g.

   b. If there are 28 grams in an ounce, how many would you need to add to your SeaPerch so that it is neutrally buoyant?

MATH CHALLENGE ANSWER SHEET

TEAM NAME: _____

TEAM SCHOOL: _____

TEAM MEMBERS' NAMES:

_____

Each fillable answer is worth one point. YOU MUST INCLUDE UNITS.

Answers:

1. (a) _____
   (b) avg = _____ median = _____ mode = _____
   (c) _____

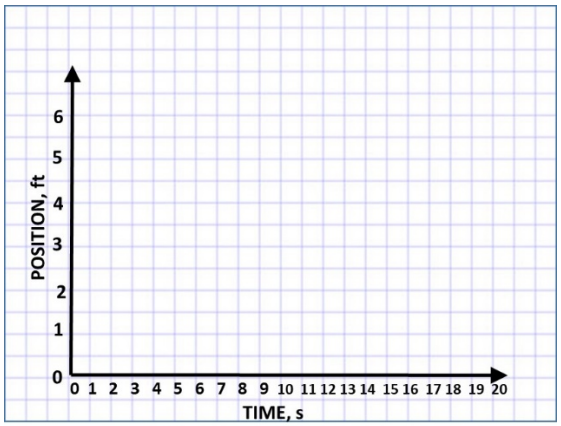
2. (a) _____
   (b) _____
   (c) _____

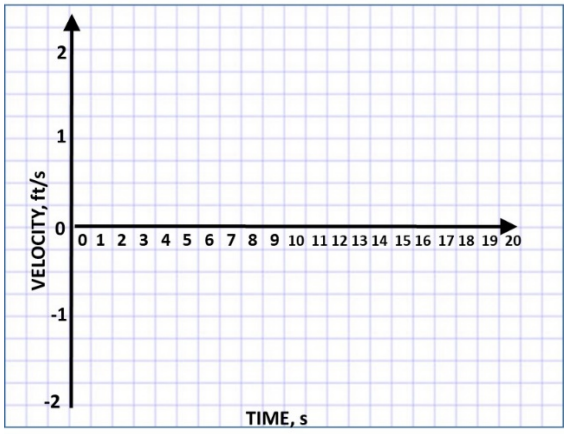
3. (a) _____
   (b) _____
   (c) _____

4. Ava _____ _____
   Liam _____ _____
   Mia _____ _____
   Noah _____ _____
   Emma _____ _____

5.

Time (s)	Position, d (ft)	Velocity, v(ft/s)
0	0	0
5	2	
8	4	
12	6	
14	6	
17	4	
19	2	
20	0	


POSITION, ft
0 1 2 3 4 5 6
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
TIME, s


VELOCITY, ft/s
2 1 0 -1 -2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
TIME, s

6. (a)_____
   (b) _____

7. (a)_____
   (b) _____
   (c) _____

8. (a)_____
   (b) _____