1. Carbon-14 decays by beta emission. Write the balanced nuclear equation.

\[
\frac{14}{6} C \rightarrow \frac{14}{7} N + {}^0 \beta \quad (\frac{14}{7} N \text{ is stable})
\]

2. If the activity of carbon-14 is \( \frac{1}{4} \) that in a living organism. How long ago did the plant die? (How many half-lives?) Two half-lives have passed since the organism died.

\[
(2)(5715 \text{ yr}) = \frac{11,430 \text{ yr}}{14} \quad \text{or use equation:}
\]

\[
\ln\left(\frac{A}{A_0}\right) = -\frac{t}{0.693} \ln\left(\frac{1}{4}\right) = \frac{11,430 \text{ yr}}{0.693}
\]

3. What are the upper and lower limits on the age of an object that can be determined by this technique? (Assume that the activity of \(^{14}\text{C} \) can be measured with an accuracy of \( \pm 1\% \).)

Upper limit discussed above: \( \approx 50,000 \text{ yr} \) (\( \approx 10 \) half-lives)

Lower limit: Assume you can accurately measure the activity if at least 1% of the \(^{14}\text{C} \) has decayed so \( \frac{A}{A_0} = 0.99 \).

\[
\ln\left(\frac{A}{A_0}\right) = -\frac{t}{0.693} \ln(0.99) = \frac{5715 \text{ yr}}{0.693} \ln(0.99) = 8.29 \text{ yr}
\]

4. A wooden object from an archaeological site is subjected to radiocarbon dating. The activity of the sample due to carbon-14 is measured to be 11.6 dps (disintegrations per second). The activity of a carbon sample of equal mass from fresh wood is 15.2 dps. What is the age of the sample?

\[
A = 11.6 \text{ dps} \\
A_0 = 15.2 \text{ dps} \\
t_{\frac{1}{2}} = 5715 \text{ yr}
\]

\[
A = \frac{A}{A_0} = \frac{11.6}{15.2} \\
\ln\left(\frac{A}{A_0}\right) = -\frac{5715 \text{ yr}}{0.693} \ln\left(\frac{11.6}{15.2}\right) = 2230 \text{ yr}
\]

Note: about \( \frac{1}{4} \) of the activity remains so \( \frac{1}{2} \) of one half-life has passed.

\[
\frac{t}{t_{\frac{1}{2}}} = \frac{1}{2}(5715 \text{ yr}) = 2858 \text{ yr} \quad \text{(pretty good estimate)}
\]
Lead-206 dating of rocks

Uranium-238 decays to lead-206 by a series of steps (see fig. 20.1). The half-life for $^{238}_{\text{U}}$ is $4.5 \times 10^9$ years. Thus, the ratio of lead-206 to uranium-238 in a uranium-containing mineral is a measure of the time since the mineral was formed. A very sensitive and precise method (mass spectrometry) exist for measuring the mass of these two isotopes in a rock sample.

1. This method assumes that all of the lead-206 in a rock came from decay of uranium-238. Is this assumption always valid? How could you tell if some of the lead-206 came from a different source? (No other radioactive decay process produces lead-206.)

2. What are the youngest rocks that could be accurately dated by this technique?

   Again assume that 1% change is measurable:
   \[ t = \frac{-4.5 \times 10^9}{0.693} \ln (0.99) = 6.5 \times 10^7 \text{ yr} \]

3. A rock contains 0.257 mg of lead-206 for every milligram of uranium-238. How old is the rock? To solve this problem, we need to know the amount of uranium-238 that was present when the rock was formed ($N_0$). This is the amount of uranium-238 present now plus the amount that decayed.

   How many moles of uranium-238 decayed to give 0.257 mg of lead-206?

   \[ \begin{align*}
   ^{238}_{\text{U}} & \rightarrow \rightarrow ^{206}_{\text{Pb}} \quad (1:1 \text{ stoichiometry}) \\
   0.257 \text{ mg Pb} & = 1.25 \times 10^{-6} \text{ mol } ^{238}_{\text{U}} \text{ decayed} \\
   1000 \text{ mg} & = 206 \text{ g} \\
   1 \text{ mol } ^{206}_{\text{Pb}} & = 1 \text{ mol } ^{238}_{\text{U}}
   \end{align*} \]

   How many moles of uranium-238 were present when the rock was formed?

   \[ \begin{align*}
   1 \text{ mg } ^{238}_{\text{U}} & = 4.2 \times 10^{-6} \text{ mol } ^{238}_{\text{U}} \text{ present now} (N) \\
   1000 \text{ mg} & = 238 \text{ g} \\
   1 \text{ mol } ^{238}_{\text{U}} & = 238 \text{ g}
   \end{align*} \]

   \[ 4.2 \times 10^{-6} \text{ mol} + 1.25 \times 10^{-6} \text{ mol} = 5.45 \times 10^{-6} \text{ mol } ^{238}_{\text{U}} \text{ originally} (N_0) \]

   Calculate the age of the rock.

   \[ \begin{align*}
   N & = 4.2 \times 10^{-6} \text{ mol} \\
   N_0 & = 5.45 \times 10^{-6} \text{ mol} \\
   t & = \frac{-t_{1/2}}{0.693} \ln \left( \frac{N}{N_0} \right) = \frac{-5.45}{4.5 \times 10^9} \ln \left( \frac{4.2}{5.45} \right) = 1.69 \times 10^9 \text{ yr}
   \end{align*} \]