IC220
Set #10:
More Computer Arithmetic (Chapter 3)

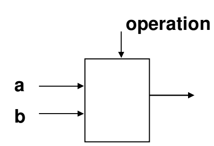
ADMIN

- **Read pages 259-262 (MIPS floating point instructions)**
- **Read 3.8**

An Arithmetic Logic Unit (ALU)

The ALU is the ‘brawn’ of the computer

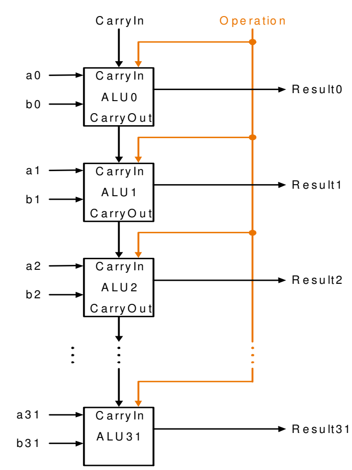
- **What does it do?**



- **How wide does it need to be?**

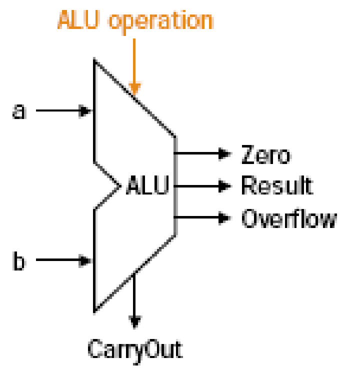
- **What outputs do we need for MIPS?**

A simple 32-bit ALU



ALU Control and Symbol

ALU Control Lines	Function
0000	AND
0001	OR
0010	Add
0110	Subtract
0111	Set on less than
1100	NOR



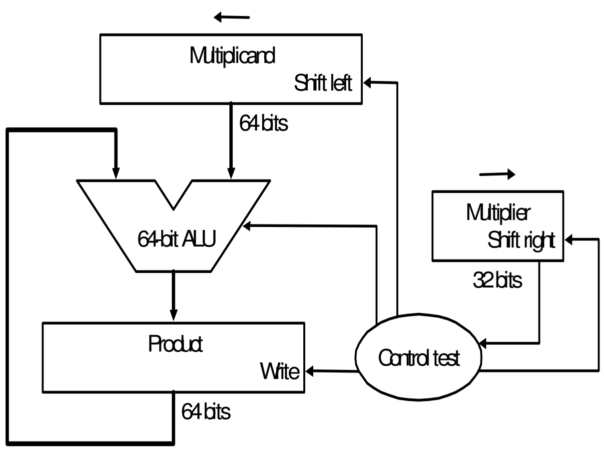
Multiplication

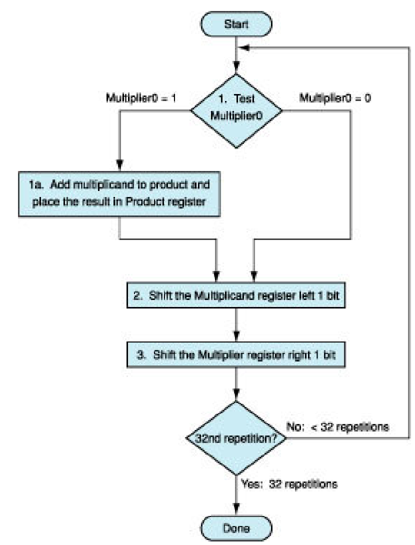
- More complicated than addition
  - accomplished via shifting and addition
- Example: grade-school algorithm

```
     0010   (multiplicand)
 __x_1011   (multiplier)
```

- Multiply m * n bits, How wide (in bits) should the product be?

Multiplication: Simple Implementation





Using Multiplication

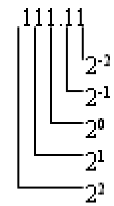
- **Product requires 64 bits**
  - **Use dedicated registers**
  - **HI – more significant part of product**
  - **LO – less significant part of product**
- **MIPS instructions**

```
mult  $s2, $s3
multu $s2, $s3
mfhi  $t0
mflo  $t1
```

- **Division**
  - **Can perform with same hardware! (see book)**

```
div  $s2, $s3        Lo = $s2 / $s3
                     Hi = $s2 mod $s3
divu $s2, $s3
```


Floating Point



- **We need a way to represent**
  - **numbers with fractions, e.g., 3.1416**
  - **very small numbers, e.g., .000000001**
  - **very large numbers, e.g., 3.15576 × 10^{23}**
- **Representation:**
  - **sign, exponent, significand:**
    - $(-1)^{sign} \times$ **significand** $\times\ 2^{exponent(some\ power)}$
  - **Significand always in normalized form:**
    - Yes:
    - No:
  - **more bits for significand gives more**
  - **more bits for exponent increases**

IEEE754 Standard

Single Precision (float): 8 bit exponent, 23 bit significand

31	30	29	28	27	26	25	24	23	22	21	20	.	.	.	9	8	7	6	5	4	3	2	1	0
S	Exponent (8 Bits)								Significand (23 bits)															

Double Precision (double): 11 bit exponent, 52 bit significand

31	30	29	28	.	.	.	21	20	19	18	17	.	.	.	9	8	7	6	5	4	3	2	1	0
S	Exponent (11 Bits)								Significand (20 bits)															

31	30	29	28	.	.	.	21	20	19	18	17	.	.	.	9	8	7	6	5	4	3	2	1	0
More Significand (32 more bits)																								

IEEE 754 – Optimizations

- **Significand**
  - **What's the first bit?**
  - **So…**
- **Exponent is "biased" to make sorting easier**
  - **Smallest exponent represented by:**
  - **Largest exponent represented by:**
  - **Bias values**
    - 127 for single precision
    - 1023 for double precision
- **Summary:** $(-1)^{sign} \times (1+\text{significand}) \times 2^{exponent - bias}$

Example:

- **Represent -5.75$_{10}$ in binary, single precision form:**

- **Strategy**
  - **Transfer into binary notation (fraction)**
  - **Normalize significand (if necessary)**
  - **Compute exponent**
    - (Real exponent) = (Stored exponent) - bias
  - **Apply results to formula**

    $(-1)^{sign} \times (1+\text{significand}) \times 2^{exponent - bias}$

Example continued:

Represent -9.75$_{10}$ in binary single precision:

- **-9.75$_{10}$ =**

111.11

2^{-2}
2^{-1}
2^{0}
2^{1}
2^{2}

- **Compute the exponent:**
  - **Remember ($2^{exponent - bias}$)**
  - **Bias = 127**

- **Formula** $(-1)^{sign} \times (1+\text{significand}) \times 2^{exponent - bias}$

31	30	29	28	27	26	25	24	23	22	21	20	.	.	.	9	8	7	6	5	4	3	2	1	0

Floating Point Complexities

- **Operations are somewhat more complicated (see text)**
- **In addition to overflow we can have "underflow"**
- **Accuracy can be a big problem**
  - **IEEE 754 keeps two extra bits, guard and round**
  - **four rounding modes**
  - **positive divided by zero yields "infinity"**
  - **zero divide by zero yields "not a number"**
  - **other complexities**
- **Implementing the standard can be tricky**

MIPS Floating Point Basics

- **Floating point registers**

  $f0, $f1, $f2, …., $f31

  Used in pairs for double precision (f0, f1) (f2, f3), …

  $f0 not always zero

- **Register conventions:**
  - **Function arguments passed in**
  - **Function return value stored in**
  - **Where are addresses (e.g. for arrays) passed?**

- **Load and store:**

  lwc1 $f2, 0($sp)

  swc1 $f4, 4($t2)

MIPS FP Arithmetic

- Addition, subtraction: add.s, add.d, sub.s, sub.d

  add.s $f1, $f2, $f3

  add.d $f2, $f4, $f6

- Multiplication, division: mul.s, mul.d, div.s, div.d

  mul.s $f2, $f3, $f4

  div.s $f2, $f4, $f6

MIPS FP Control Flow

- Pattern of a comparison: c.___.s (or c.___.d)

```
c.lt.s $f2, $f3
c.ge.d $f4, $f6
```

- Where does the result go?

- Branching:

```
bc1t      label10
bc1f      label20
```


Example #1

- Convert the following C code to MIPS:

```
float max (float A, float B) {
  if (A <= B) return A;
  else        return B;
}
```


Example #2

EX: 3-21 …

- Convert the following C code to MIPS:

```
void setArray (float F[], int index,
               float val) {
  F[index] = val;
}
```

Chapter Three Summary

- Computer arithmetic is constrained by limited precision
- Bit patterns have no inherent meaning but standards do exist
  - two's complement
  - IEEE 754 floating point
- Computer instructions determine "meaning" of the bit patterns
- Performance and accuracy are important so there are many complexities in real machines (i.e., algorithms and implementation).

- We are (almost!) ready to move on (and implement the processor)

Chapter Goals

- Introduce 2's complement numbers
  - Addition and subtraction
  - Sketch multiplication, division
- Overview of ALU (arithmetic logic unit)
- Floating point numbers
  - Representation
  - Arithmetic operations
  - MIPS instructions