2.3 Valid and Invalid Arguments
Valid Arguments

Definition

1. An argument (argument form) is a sequence of statements (statement forms).
2. All statements in an argument, except the final one, are called premises (or assumptions or hypothesis).
3. The final statement is called the conclusion.
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<td>5. An argument is valid means that its form is valid.</td>
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Testing an Argument Form for Validity

Fact

1. Identify the premises and conclusion of the argument form.
2. Construct a truth table showing the truth values of all the premises and the conclusion.
3. A row of the truth table in which all the premises are true is called a critical row.
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2. Construct a truth table showing the truth values of all the premises and the conclusion.
3. A row of the truth table in which all the premises are true is called a critical row.
   1. If there is a critical row in which the conclusion is false, then the argument is invalid.
   2. If the conclusion in every critical row is true, then the argument form is valid.
Modus Ponens

Fact

- An argument form consisting of two premises and a conclusion is called a syllogism.
- The most famous example is modus ponens ("mood that affirms"): If \( p \rightarrow q, \ p :. \ q \)

```
<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \rightarrow q</th>
<th>p</th>
<th>q</th>
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Example

Here is a modus ponens argument: If it snows more than 2” then the Naval Academy closes. It snowed more than 2”.

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**Modus tollens** ("mood that denies") has the form

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Additional rules of inference

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1. **Generalization**: \( p \vdash p \lor q \) or
2. **Specialization**: \( p \land q \vdash p \lor q \) or \( p \land q \vdash q \).
3. **Elimination**: \( p \lor q, \neg q \vdash p \) or \( p \lor q, \neg p \vdash q \).
4. **Transitivity**: \( p \rightarrow q, q \rightarrow r \vdash p \rightarrow r \).
5. **Division into Cases**: \( p \lor q, p \rightarrow r, q \rightarrow r \vdash r \).
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Give examples of each of the above rules of inference.
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Give examples of each of the above rules of inference.
We discuss two fallacies:

1. **Converse Error**

Example: If Midshipmen X cheats on the test then Midshipmen X sits in the last row. Midshipmen X sits in the back row. ∴ Midshipmen X cheats on the test.

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Example: If Midshipmen X studies for the test then Midshipmen X gets a high score on the test. Midshipmen X does not study for the test. ∴ Midshipmen X gets a low score on the test.
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If you can show that the supposition that statement $p$ is false leads logically to a contradiction, then you can conclude that $p$ is true. Symbolically

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The logician Raymond Smullyan describes an island containing two types of people: knights who always tell the truth and knaves who always lie. You visit the island and are approached by two natives who speak to you as follows:

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B says: A and I are of opposite type.

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