Intro to Probabilistic Robotics

Intelligent behavior can only be gained in robots if they are responding to inputs from their sensors. In other words, they have to have some understanding of the world in order to behave appropriately. Unfortunately, sensors are unreliable; things go wrong frequently, meaning that depending too much on any given reading is a recipe for failure.

So, if we can’t trust our sensors, what hope is there? Imagine using a rangefinder, which can tell distance from the robot to a wall. On five consecutive readings from a stationary robot, it tells us the wall is 105 cm, 92 cm, 506 cm, 103 cm, and 98 cm away. Our sensor is clearly jumping all over the place. But, is there truly nothing we can learn from these readings? Of course not. In fact, it is likely obvious to you that the wall is likely somewhere around 100 cm away. You might even say that it is probable that the wall is close to 100 cm away.

We can formalize this intuition by using the mathematical field of probability. Eventually we’ll understand this well enough to do some exciting things, like mapping, even with unreliable sensors.

Inferring Knowledge about the World

Let’s start with a very simple example (scenario cribbed from Thrun et al. “Probabilistic Robotics”). We’ll learn some things about probability along the way.

Suppose we have a robot, capable of opening a door. It also has a sensor, which tells it if the door is open or closed. The notation we’ll use is as follows: \( o \) will denote our observation, and \( x \) will denote our “state”, meaning the condition of reality (in this case, our state is if the door is open or closed).

Now, this sensor is unreliable (or “noisy”). If the door is closed, it gets this wrong, and observes it as open 20% of the time. We can denote this as follows:

\[
p(o = \text{closed} | x = \text{closed}) = 0.8
\]

\[
p(o = \text{open} | x = \text{closed}) = 0.2
\]

We read this as “the probability that the observation is closed, given that the door is closed, is 0.8.” This is known as a conditional probability, because the probability of the thing before the vertical line being true is conditioned upon the thing after the vertical line being true.

Let’s go back to our sensor. If the door is actually open, it gets this wrong, and observes it as open 20% of the time. We can denote this as follows:

\[
p(o = \text{closed} | x = \text{open}) = 0.4
\]

\[
p(o = \text{open} | x = \text{open}) = 0.6
\]

What we are interested in finding out is given our readings, what is the likelihood that the door is open? In other words, we want:

\[
p(x = \text{open} | o).
\]

Going from \( p(a|b) \) to \( p(b|a) \) is a big, useful deal, which deserves its own box.

| It is a fundamental fact of probability that \( p(x, y) \) (meaning “the probability of \( x \) and \( y \)”) equals \( p(x|y)p(y) \). That lets us do the following derivation: |
| --- |
| \( p(x, y) = p(y, x) \) |
| \( p(x|y)p(y) = p(y|x)p(x) \) |
| \( p(x|y) = \frac{p(y|x)p(x)}{p(y)} \) |

This is known as Bayes’ Rule. \( p(x) \) is known as the prior, and \( p(x|y) \) is known as the posterior.
Let’s try this. Let’s say that at the first timestep, our robot’s sensor takes a reading, and observes the door is open ($o = \text{open}$). What is the most likely state $x$?

We only have two possible states, so what we want to calculate is $p(x = \text{open}|o = \text{open})$ and $p(x = \text{closed}|o = \text{open})$.

Let’s apply Bayes’ rule.

\[
p(x = \text{open}|o = \text{open}) = \frac{p(o = \text{open}|x = \text{open})p(x = \text{open})}{p(o = \text{open})}
\]

\[
= \frac{0.6p(x = \text{open})}{p(o = \text{open})}
\]

\[
p(x = \text{closed}|o = \text{open}) = \frac{p(o = \text{open}|x = \text{closed})p(x = \text{closed})}{p(o = \text{open})}
\]

\[
= \frac{0.2p(x = \text{open})}{p(o = \text{open})}
\]

We still have two unknowns. The first is our priors $p(x = \text{open})$ and $p(x = \text{closed})$. Now, suppose we have some knowledge like that the door is closed 90% of the time; this is where this human knowledge could come into play, by making $p(x = \text{open}) = 0.1$. This would skew the results such that one unreliable sensor reading wouldn’t overcome our prior belief that the door is probably closed. In this case, we’ll assume that our belief is that the door is open about half the time.

\[
p(x = \text{open}|o = \text{open}) = \frac{0.6 \times 0.5}{p(o = \text{open})}
\]

\[
p(x = \text{closed}|o = \text{open}) = \frac{0.2 \times 0.5}{p(o = \text{open})}
\]

As always, “there’s an XKCD for that,” even for the use of a prior in statistics. See Figure 1. People who do statistics with priors using Bayes’ rule are known as “Bayesians,” while those who don’t are called “frequentists.” The holy war between the two rivals that of the operating systems war, or even the vi vs. Emacs war.

OK, so we’re nearly there. Now, we just have to fill in $p(o = \text{open})$, and we’ll have calculated our posteriors. Sometimes we’ll know that number, and sometimes we won’t. Fortunately, we never have to care. Because the door MUST be either open or closed, $p(x = \text{open}|o) + p(x = \text{closed}|o) = 1$. See why? So, we can say that

\[
\frac{0.6 \times 0.5}{p(o = \text{open})} + \frac{0.2 \times 0.5}{p(o = \text{open})} = 1
\]

Work the math through, and we get that $p(o = \text{open}) = .4$. So,

\[
p(x = \text{open}|o = \text{open}) = \frac{0.6 \times 0.5}{0.4} = .75
\]

\[
p(x = \text{closed}|o = \text{open}) = \frac{0.2 \times 0.5}{0.4} = .25
\]

So, we can conclude there is a 75% chance the door is actually open, given our sensor reading.

Now, we’ll have the robot take the action of opening the door. We’ll denote the state before the action as $x_{t-1}$ and the state after as $x_t$ (because they’re the states at timesteps $t-1$ and $t$, respectively). We’ll notate the action with $u$. Like our sensor, our actuator is “noisy,” meaning it isn’t always effective. In fact, we have the following probabilities:
So, we took a sensor reading, determined there was a 75% chance of it being open, and now we’re going to push on the door. What is the probability the door is open?

For this, we’ll use the following fact:

$$p(a) = \sum_b p(a|b)p(b).$$

So,

$$p(x_t = \text{open}|u = \text{push}, x_{t-1} = \text{open}) = p(x_t = \text{open}|u = \text{push}, x_{t-1} = \text{open})p(x_{t-1} = \text{open})$$

$$+ p(x_t = \text{open}|u = \text{push}, x_{t-1} = \text{closed})p(x_{t-1} = \text{closed})$$

$$= 1 \times 0.75 + 0.8 \times 0.25$$

$$= 0.95$$

So, there is now a 95% chance that the door is open.
This algorithm will guide us for a while. We start with some belief about the world, we take an action, which changes our belief, we take a sensor reading, which changes our belief again. We then repeat. This is known as a Bayes Filter.