1. (30 Points) Write regular expressions for the following languages:

   (a) The language of strings over \{a, b\} that start and end with the same character. E.g. “abaa” or “b”, but not “abb”.

   (b) The language of all comma-separated non-empty strings over \{a, b\}. E.g. “a, aba, bb” or “abb”, but not “ab, , b” or “b, ab, ,”.

   (c) The language of strings over \{a, b, c\} in which every c is followed immediately by an a.

2. (35 Points) Consider the following algorithm, which proves that there is no finite automaton accepting \(L\), the language over the alphabet \(\Sigma = \{a, b, c\}\) whose length is prime.

   **Input:** DFA \(M = (Q, \Sigma, \delta, s, W)\)
   
   **Output:** string \(w'\) such that either \(w' \in L\) but \(w' \notin L(M)\), or \(w' \notin L\) but \(w' \in L(M)\).

   (a) let \(p\) be the first prime strictly greater than \(|Q| + 1\)
   
   (b) let \(w = c^p\) (Note: \(w \in L\) and \(|w| > |Q|\), so if \(w \notin L(M)\) we can apply the Pumping Lemma)
   
   (c) if \(w \notin L(M)\)
   then set \(w' = w\)
    else
      i. let \(x, y, z\) be the strings guaranteed to exist by the Pumping Lemma Version 2.0. (i.e. \(w = xyz, y \neq e, |xy| \leq |Q|\) and \(xy^kz \in L(M)\) for all \(k\))
      ii. set \(w' = xy^{(p-|y|)}z\) (Note: \(|w| = |xz| + |y|(p-|y|) = (p-|y|)(1+|y|) which, since both factors are greater than 1, is not prime.)

   What string is returned by this algorithm when the following machine is given to it as input:

   ![DFA Diagram]

   Show your work!

3. (35 Points) Give a Pumping Lemma based proof that the language \(L\) of all strings over \{a, b\} in which the number of as is exactly twice the number of bs is not regular, i.e. is not accepted by any finite automaton. (Do it on back!)