1. (30 Points) Write regular expressions for the following languages:

   (a) The language of strings over \( \{a, b\} \) that start and end with the same character. E.g. “abaa” or “b”, but not “abb”.

   (b) The language of all comma-separated non-empty strings over \( \{a, b\} \). E.g. “a,aba,bb” or “abb”, but not “ab,,b” or “b,ab,”.

   (c) The language of strings over \( \{a, b, c\} \) in which every \( c \) is followed immediately by an \( a \).

2. (35 Points) Consider the following algorithm, which proves that there is no finite automaton accepting \( L \), the language over the alphabet \( \Sigma = \{a, b, c\} \) whose length is prime.

   **Input:** DFA \( M = (Q, \Sigma, \delta, s, W) \)
   
   **Output:** string \( w' \) such that either \( w' \in L \) but \( w' \notin L(M) \), or \( w' \notin L \) but \( w' \in L(M) \).

   (a) let \( p \) be the first prime strictly greater than \(|Q| + 1\)
   
   (b) let \( w = c^p \) (Note: \( w \in L \) and \(|w| > |Q| \), so if \( w \notin L(M) \) we can apply the Pumping Lemma)

   (c) if \( w \notin L(M) \)
      
      then set \( w' = w \)
   
      else
      
      i. let \( x, y, z \) be the strings guaranteed to exist by the Pumping Lemma Version 2.0. (i.e. \( w = xyz \), \( y \neq \epsilon \), \(|xy| \leq |Q| \) and \( xy^kz \in L(M) \) for all \( k \).)
      
      ii. set \( w' = xy^{(p-|y|)}z \) (Note: \(|w| = |xz| + |y|(p-|y|) = (p-|y|)(1+|y|) \) which, since both factors are greater than 1, is not prime.)

   What string is returned by this algorithm when the following machine is given to it as input:

   ![DFA Diagram]

   Show your work!
3. (35 Points) Give a Pumping Lemma based proof that the language $L$ of all strings over $\{a, b\}$ in which the number of $a$s is exactly twice the number of $b$s is not regular, i.e. is not accepted by any finite automaton. (Do it on back!)