Section 13.3: Arclength of curves

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$s = v t$
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\[
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Arclength

Let \( \mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle \) be a parameterized path (aka curve). With your group, write down an expression for the speed at time \( t \). Call your function \( v(t) \), ... and notice it is NOT a vector!

Definition

The arclength of \( \mathbf{r}(t) \) from \( t = a \) to \( t = b \) is the distance traveled along the path \( \mathbf{r}(t) \) from time \( a \) to time \( b \).
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QUESTION: How do you compute arclength?
Aside

What is the area of the shape below (it’s fine with me if you assume it’s a parabola).
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Discussion:

- Why is the area not simply given by AREA=BASE×HEIGHT?
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- Why is the area not simply given by \( \text{AREA} = \text{BASE} \times \text{HEIGHT} \)?
- Think of other rules from mathematics or the sciences of the form \( C = A \times B \). When do you need an integral to compute \( C \)?
Arc-length formula

The arclength of $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ from $t = a$ to $t = b$ is:

$$s = \int_{a}^{b} \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2} \, dt.$$
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s = \int_{a}^{b} |\mathbf{r}'(t)| \, dt = \int_{a}^{b} \mathbf{v}(t) \, dt.
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In the rest of the course, it is useful to define the differential

\[
ds = \mathbf{v}(t) \, dt = \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2} \, dt.
\]

Which we think of as “a little bit of arclength”. Then we can write \( s = \int ds \).
Example

With your group, compute the arc-length from \( t = 0 \) to \( t = 3\pi \) for the helix parameterized by

\[
x = 3 \cos(2t), \quad y = 3 \sin(2t) \quad z = 8t
\]