14.1 Functions of several variables

Definition

- A function of two variables, defined on a domain $D$ in the plane, is a rule $f$ that assigns to each point $(x, y)$ in $D$ a unique real number, denoted $f(x, y)$. 

- A function of three variables, defined on a domain $D$ in space, is a rule $f$ that assigns to each point $(x, y, z)$ in $D$ a unique real number, denoted $f(x, y, z)$. 

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Per the definition on the previous slide, which of the following defines a “function of three variables”?

(A) The parameterized curve \( \mathbf{r}(t) = \langle t, t^2, t^3 \rangle \) (the twisted cubic).

(B) The equation \( x^2 + y^2 - z^2 = 1 \) cutting out a hyperboloid of one sheet.

(C) To each point in this classroom, assign the temperature.

(D) More than one of these.
What is the largest possible domain of the function

\[ f(x, y) = \sqrt{25 - x^2 - y^2} \]

(A) \( -5 \leq x \leq 5, \ -5 \leq y \leq 5. \)

(B) \( x^2 + y^2 \leq 25. \)

(C) The closed (i.e. includes its boundary) disk of radius 5 centered at the origin.

(D) More than one of these.
Which of the following best describes the points in the $xy$-plane for which $f(x, y) = \pm 1$ for the function

$$f(x, y) = \frac{y}{\sqrt{x - y^2}}.$$ 

(A) A circle of radius 1 centered at the origin.

(B) A quadric surface in $\mathbb{R}^3$ which is a cylinder.

(C) A parabola in the $xy$-plane opening in the $x$ direction with vertex at the origin.

(D) None of these.
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- The corresponding **level set** or **level curve** is the projection of the contour curve into the \( xy \)-plane. In other words, it is the curve defined by \( k = f(x, y) \) in the \( xy \)-plane.
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- A drawing depicting a collection of level curves for many values of $k$ is called a **contour map**.
Examples in Mathematica.