Name: ______________________________ Class Time: 2 days

**Purpose**: To introduce the concept of double and triple integrals, computed as *iterated integrals*.

**Procedure**: Work on the following problems. Bring your ideas and solutions to class to discuss.

Let $R$ denote the region in $\mathbb{R}^2$ described by $1 \leq x \leq 3$ and $-3 \leq y \leq 5$.

**Problem 1.** Sketch a picture of the region $R$.

**Problem 2.** The *double integral* of $f(x, y) = x^2y + 1$ over the region $R$ is denoted by

$$\iint_R f(x, y) \, dA$$

where $dA$ means "a little bit of area in 2-dimensions". However, the integral is *computed* by doing the *iterated integral*

$$\int_1^3 \left( \int_{-3}^5 f(x, y) \, dy \right) \, dx.$$

Compute the iterated integral.

**Problem 3.** What happens if you instead compute the iterated integral $\int_{-3}^5 \int_1^3 f(x, y) \, dx \, dy$?

**Problem 4.** Describe in words (as precisely as possible) what the value of the double integral above represents in geometry.

**Problem 5.** Now let $F : \mathbb{R}^3 \to \mathbb{R}$ be defined by $F(x, y, z) = xyz^2$. Compute the *triple integral*

$$\iiint_\Omega F(x, y, z) \, dV$$

over the region $\Omega$ defined by $0 \leq x \leq 1$, $1 \leq y \leq 2$, and $-1 \leq z \leq 4$.

**Problem 6.** What does $dV$ mean in the integral above?

**Problem 7.** Describe in words (as precisely as possible) what the value of the triple integral above represents in geometry. Can you think of a "real life" example when this triple integral has meaning?