Day 2 Homework

Let \( T \) denote the region in \( \mathbb{R}^2 \) bounded by the lines \( y = 0 \), \( y = x \), and \( x = 2 \). Moreover, let \( g : \mathbb{R}^2 \to \mathbb{R} \) be the function defined by \( g(x, y) = y\sqrt{x^2 - y^2} \).

**Problem 8.** Sketch the region \( T \). Is \( T \) in the domain of \( g \)?

**Problem 9.** Describe in words (as precisely as possible) what the value of the double integral

\[
\iint_T g(x, y) \, dA
\]

represents in geometry.

**Problem 10.** Write down an iterated integral for \( \iint_T g(x, y) \, dA \) in which you integrate *first* with respect to \( y \) and *then* with respect to \( x \).

**Problem 11.** This time, write down an iterated integral for \( \iint_T g(x, y) \, dA \) in which you integrate first \( x \), then \( y \).

**Problem 12.** Which of Problems 10 or 11 is computationally easier? Why? Compute the value of the double integral \( \iint_T g(x, y) \, dA \).

Let \( E \) denote the region in \( \mathbb{R}^3 \) which lies above the triangle \( T \) in the \( xy \)-plane but below the graph of the surface \( z = \sqrt{y} \).

**Problem 13.** Suppose that the region \( E \) is made up of a material whose density is not constant. In fact, assume that the density of \( E \) is given by the function \( \delta(x, y, z) = z\sqrt{x^2 - y^2} \) in kg/cm\(^3\). Assume that the units on the \( x \), \( y \), and \( z \) axis are “centimeters” and compute the mass (in kg) of the region \( E \).

**Problem 14.** Let \( a \) and \( b \) be any real numbers on the \( x \)-axis (assume that \( a < b \)). Compute the value of the Calc I integral: \( \int_a^b 1 \, dx \). What does this integral tell you about the interval \([a, b]\)?

**Problem 15.** Let \( D \) denote a bounded region (i.e. \( D \) has finite area) in the \( xy \)-plane. What does the value of the double integral \( \iint_D 1 \, dA \) tell you about \( D \)?

**Problem 16.** Let \( E \) denote a bounded region (i.e. \( E \) has finite volume) in \((x, y, z)\)-space. What does the value of the triple integral \( \iiint_E 1 \, dV \) tell you about the region \( E \)?