Knowledge Demonstration Opportunity 1: SM221P, Calculus III

Name: _______________________

13 September 2017

Read all of the following information before starting:

- You are allowed pencils, pens, your TI-36X calculator, and your wits. That is all. In particular, no computers, notes, books, smartphones, iPads, or pocket-sized hobbits.
- To receive full credit, justify your work clearly and in order. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your “final” answer is correct).
- Use sentences to explain your reasoning. Please keep written answers brief; and simultaneously clear!
- Box or otherwise indicate your final numeric answers.
- Good luck!

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible</th>
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<td>Total</td>
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Problem 1 (10 points). Find the equation of the plane through the points (1, 2, 3), (4, 5, 6) and (7, 9, 11).
Problem 2 (10 points). Circle the correct answer for each multiple choice question below. Show any computations you need to make in the white space for each question.

(a) Find all point(s) of intersection in $\mathbb{R}^3$ of the line defined by $x = 1 + t, y = t, z = 1 - t$ with the paraboloid $z = x^2 + y^2$.

(i) $(1, 0, 1)$;
(ii) $(-1/2, -3/2, 5/2)$;
(iii) Choices (i) and (ii) are both correct and they are the only correct answers;
(iv) Choices (i) and (ii) are both correct, but there are still more correct answers not listed;
(v) none of the above.

(b) Which of the following statements is always true for any 3-dimensional vectors $a$, $b$, and $c$?

(i) If $a \times b = a \times c$, then $b = c$;
(ii) If $a \cdot b = a \cdot c$, then $b = c$;
(iii) If $a$ is a scalar multiple of $b$, then $a \times b = 0$;
(iv) If $a$ is a scalar multiple of $b$, then $a \cdot b = 0$;
(v) choices (ii) and (iii) are true, but (i) and (iv) are false;
(vi) none of the above are true.

(c) Suppose that $u$, $v$, and $w$ are 3-dimensional vectors with the following properties:

- the angle between $u$ and $v$ is obtuse
- $u$ is perpendicular to $w$.

Then the scalar $u \cdot (2021w - 4v)$ is

(i) positive;
(ii) negative;
(iii) zero;
(iv) there is not enough information to determine whether (i), (ii), or (iii) is true.
Problem 3 (10 points). In class, we said that a **quadric surface** was a 2-dimensional object in $\mathbb{R}^3$ which is cut out by a degree 2 polynomial equation of the form:

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = G.$$  

However, we could have also allowed “mixed” degree 2 terms of the form: $xy$, $xz$, or $yz$ (in fact, our textbook does allow this). So, in this spirit...

Let’s consider the quadric surface in $\mathbb{R}^3$ cut out by the equation

$$y^2 = xz$$

In the spaces provided below, sketch the indicated traces.

(a) Sketch traces for the values $x = 0$, $x = 1$ and $x = -1$ below:

<table>
<thead>
<tr>
<th>$x = 0$</th>
<th>$x = 1$</th>
<th>$x = -1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Sketch" /></td>
<td><img src="image2.png" alt="Sketch" /></td>
<td><img src="image3.png" alt="Sketch" /></td>
</tr>
</tbody>
</table>

(b) Sketch traces for the values $y = 0$, $y = \pm 1$ and $y = \pm 2$ below:

<table>
<thead>
<tr>
<th>$y = 0$</th>
<th>$y = \pm 1$</th>
<th>$y = \pm 2$</th>
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<tbody>
<tr>
<td><img src="image4.png" alt="Sketch" /></td>
<td><img src="image5.png" alt="Sketch" /></td>
<td><img src="image6.png" alt="Sketch" /></td>
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(c) Identify which picture corresponds to the quadric surface cut out by $y^2 = xz$. 
Problem 4 (10 points). Consider the planes in $\mathbb{R}^3$ defined by the equations

\[ 5x + 2y - z = 6 \quad \text{and} \quad 3x + z = 6. \]

Show that the following are parametric equations for the line of intersection of the two planes:

\[ x = 1 - t \quad \quad y = 2 + 4t \quad \quad z = 3 + 3t. \]