Read all of the following information before starting:

- You are allowed pencils, pens, your TI-36X calculator, and your wits. That is all. In particular, *no computers, notes, books, smartphones, iPads, or pocket-sized hobbits.*
- To receive full credit, justify your work clearly and in order. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your “final” answer is correct).
- Use sentences to explain your reasoning when necessary. Keep written answers brief and simultaneously clear!
- Box or otherwise indicate your final numeric answers.
- Good luck!

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<thead>
<tr>
<th>Problem</th>
<th>Possible</th>
<th>Score</th>
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<tr>
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Problem 1 (10 points). Consider the conservative vector field

$$\mathbf{F} = \langle 4x^3yz^4 + ze^y, x^4z^4 + xze^y, 4xz^3 + xe^y \rangle.$$ 

(a) Find a potential function \( f(x, y, z) \) for which \( \nabla f = \mathbf{F} \).

(b) Compute the value of the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) for the curve \( C \) parameterized by

\[
x = \arctan \left( \frac{\ln(t + 1)}{\ln(2)} \right), \quad y = t^3 - t, \quad z = \sin(4\pi t) - 1, \quad (0 \leq t \leq 1)
\]
Problem 2 (10 points). Clearly indicate your answer to each of the multiple choice questions below.

(a) Below is a picture of a vector field $\mathbf{F}$ and three paths (the three sides of the right triangle). Note the indicated orientation on each path, and list the paths in order of increasing value of the work done by $\mathbf{F}$ along each path (i.e. list from smallest value of work to largest value of work).

(i) hypotenuse, right leg, bottom leg
(ii) hypotenuse, bottom leg, right leg
(iii) right leg, hypotenuse, bottom leg
(iv) right leg, bottom leg, hypotenuse
(v) bottom leg, hypotenuse, right leg
(vi) bottom leg, right leg, hypotenuse

(b) Which of the following vector fields produces the largest flux out of the unit sphere centered at the origin in $\mathbb{R}^3$?

(i) $\mathbf{F}_1 = (e^z + x^3)i + e^xj + y^3k$
(ii) $\mathbf{F}_2 = (z^2 + \cos y)i - y^3j + x^3y^3k$
(iii) $\mathbf{F}_3 = z^2i - (x^2 + z^2)j + (z^3 + zy^2)k$
(iv) $\mathbf{F}_4 = (x^4 - y^4)i - (z^4 - 2x^3y)j + (y^4 - 2x^3z)k$

(c) Let $\mathbf{F} = (\frac{1}{2}x^2y + e^x)i + (x - \frac{1}{4}xy^2)j$. Choose the simple closed curve $C$ below which would maximize the value of the work \( \int_C \mathbf{F} \cdot d\mathbf{r} \). For reference, each image below depicts the portion of the $xy$-plane with $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$. 
Problem 3 (10 points). Let $P$ denote the surface with upward orientation consisting of the part of the paraboloid $z = 4 - x^2 - y^2$ above the rectangle $0 \leq x \leq 1$ and $0 \leq y \leq 1$ in the $xy$-plane. Compute the flux $\iint_P \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = (x^2, 2y, z)$. 
Problem 4 (10 points). Let $G = \langle y^2z - z^2 e^y, xe^z, y + xz^2 \rangle$ and compute the flux $\iint_S \text{curl}(G) \cdot dS$ where $S$ is the part of the sphere $x^2 + y^2 + z^2 = 4$ with $x \geq 0$, and oriented towards the positive $x$-direction.