Question: Given a function \( f(x, y) \), how do we find its maximum and minimum (aka extreme) values? How do we find the points \((a, b)\) in the domain where they occur? How do we recognize local extrema from global or absolute extrema?
Remember Calc I: $y = f(x)$.

- Local maxima and minima occur only when $y' = 0$. To know which, *have a second derivative test*.
- This worked because “$y' = 0$” implies “the tangent line is horizontal”.
- Sometimes when $y' = 0$, we don’t get a max/min, but a “saddle” (e.g. $y = x^3$).
Theorem

If both partial derivatives of $f$ exist at $(a, b)$ and $f$ has a local maximum or minimum at $(a, b)$, then

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0.$$
Theorem

If both partial derivatives of $f$ exist at $(a, b)$ and $f$ has a local maximum or minimum at $(a, b)$, then

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0.$$ 

Question: How does this relate to the formula for the equation of the tangent plane?
Example

With your group, claim a spot at the board and then find all points in $\mathbb{R}^2$ where

$$f(x, y) = (2x^2 + 3y^2)e^{-x^2-y^2}$$

has a horizontal tangent plane,
a.k.a. has $\nabla f = \mathbf{0}$,
a.k.a. has $f_x = 0$ and $f_y = 0$. 