Introduction to Quadric Surfaces.

Purpose: To review conic sections, and extend their properties to three dimensions.

Procedure: Work on the following problems outside of class, on your own. Bring your ideas and solutions to class to discuss.

Sketch graphs of the following conic sections on the indicated plane.

Problem 1. In the $xy$-plane, draw the graph of $\frac{x^2}{4} + y^2 = 1$.

Problem 2. In the $xy$-plane, draw the graph of $\frac{x^2}{4} + y^2 = 2$.

Problem 3. In the $xy$-plane, draw the graph of $\frac{x^2}{4} + y^2 = 5$.

Problem 4. In the $xz$-plane, draw the graph of $\frac{x^2}{4} - \frac{z^2}{4} = 1$.

Problem 5. In the $xz$-plane, draw the graph of $\frac{x^2}{4} - \frac{z^2}{4} = 0$.

Problem 6. In the $xz$-plane, draw the graph of $\frac{x^2}{4} - \frac{z^2}{4} = -3$.

Problem 7. In the $yz$-plane, draw the graph of $y^2 - \frac{z^2}{4} = 1$.

Problem 8. In the $yz$-plane, draw the graph of $y^2 - \frac{z^2}{4} = 0$.

Problem 9. In the $yz$-plane, draw the graph of $y^2 - \frac{z^2}{4} = -3$.

Definition 1. A quadric surface is a 2-dimensional surface in 3-dimensional space defined by an equation of the form:

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = G$$

for some scalars $A, B, \ldots, G$ (where at least one of $A, B,$ or $C$ is not zero). That is, they are defined by quadratic polynomials (aka degree 2 polynomials). In this sense, they are the 3-dimensional generalization of the conic sections in 2-dimensions.

Consider the quadric surface defined by the equation

$$\frac{x^2}{4} + y^2 - \frac{z^2}{4} = 1 \quad (1)$$

Problem 10. What are the values of $A, B, \ldots, G$ for the quadric surface defined by Equation (1)?

Problem 11. Can you observe any connections between the conic sections in Problems 1–9 and the quadric surface defined by Equation (1)? List at least 2 of your observations you would like to share with your group and/or the class.