Knowledge Demonstration Opportunity 1: SM221, Calculus III

Name: ____________________________

5 February 2018

Read all of the following information before starting:

• You are allowed pencils, pens, your TI-36X calculator, and your wits. That is all. In particular, no computers, notes, books, smartphones, iPads, or pocket-sized hobbits.

• To receive full credit, justify your work clearly and in order. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your “final” answer is correct).

• Use sentences to explain your reasoning. Please keep written answers brief; and simultaneously clear!

• Box or otherwise indicate your final numeric answers.

• Good luck!

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Problem 1 (10 points). Let \( f(x, y) = 19x^2 - 3x^2y - 7xy^2 + 14y^3 \).

(a) Compute the gradient \( \nabla f(x, y) \)

(b) Compute the directional derivative \( D_v f(1, -1) \) in the direction \( v = \langle -\frac{3}{5}, \frac{4}{5} \rangle \).

(c) Write an equation for the tangent plane to the graph \( z = f(x, y) \) at the point \( (1, -1, 1) \).
Problem 2 (10 points). Answer each of the following multiple choice questions.

(a) Suppose that $T(x, y, z) = \frac{4x + 2y^2}{z+1}$ represents the temperature in degrees celcius in the first octant (aka where $x > 0$, $y > 0$, and $z > 0$) in 3D space. At the point $(2, 1, 2)$ which of the following vectors is in parallel to the direction of greatest instantaneous change in the temperature?

(i) $\langle 4, 4, 10 \rangle$;
(ii) $\langle 4, 4, -10 \rangle$;
(iii) $\langle 4, 4, -10/3 \rangle$;
(iv) $\langle 4, 4, 10/3 \rangle$;
(v) $\langle -4/3, -4/3, -10/9 \rangle$.

(b) Which of the following best describes the path in 2D space which is parameterized by the function below?

$r(t) = (5 \cos(t^2), 5 \sin(t^2)), \quad t \geq 0$

(i) a spiral whose radius increases as $t$ increases;
(ii) an ellipse whose axes increase as $t$ increases;
(iii) a circle around which we move faster and faster as $t$ increases;
(iv) a quarter of a circle in the first quadrant;
(v) a parabola on which we move back and forth.

(c) The first picture below is the contour plot of a function $z = f(x, y)$. The next two plots are graphs of functions $x(t)$ and $y(t)$. Use them to decide if the derivative $\frac{dz}{dt}|_{t=1/2}$ is

(i) positive;
(ii) negative;
(iii) approximately zero;
(iv) can’t tell from the information given.
**Problem 3** (10 points). In class we only discussed quadric surfaces whose defining equations had no terms of the form: $xy$, $yz$, or $xz$. However, because each of these monomials is of degree two, they still can be used to define quadric surfaces. In this problem, consider the quadric surface defined by the equation

$$y^2 = xz$$

(a) Sketch traces for the values $x = 0$, $x = 1$ and $x = −1$ below:

(b) Sketch traces for the values $y = 0$, $y = ±1$ and $y = ±2$ below:
(c) Identify which picture corresponds to the quadric surface cut out by $y^2 = xz$. 
Problem 4 (10 points). Suppose that an ion travels through an electric field in 3D space with acceleration at time $t \geq 0$ given by the vector function

$$\mathbf{a}(t) = (\sin(t), 2 \cos(t), \sin(6t)).$$

(a) If its initial velocity is $\mathbf{v}(0) = \langle -1, 0, -1/6 \rangle$ and its initial position is the origin, find parametric expressions for the velocity $\mathbf{v}(t)$ and position $\mathbf{r}(t)$.

(b) Write down, but do not evaluate, an integral which computes the arc-length of the particles path from time $t = 0$ to time $t = 3\pi$. 