Knowledge Demonstration Opportunity 2: SM221, Calculus III

Name: ______________________

3 October 2018

Read all of the following information before starting:

• You are allowed pencils, pens, your TI-36X calculator, and your wits. That is all. In particular, no computers, notes, books, smartphones, iPads, or pocket-sized hobbits.

• To receive full credit, justify your work clearly and in order. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your “final” answer is correct).

• Use sentences to explain your reasoning. Please keep written answers brief; and simultaneously clear!

• Box or otherwise indicate your final numeric answers.

• Good luck!

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Problem 1 (10 points). Consider the function \( f(x, y) = x^2 y^2 - xy^3 + y \).

(a) Find all critical points of \( f \).

(b) Determine whether each critical point you found in (a) corresponds to a local maximum, a local minimum, or a saddle point.
Problem 2 (10 points). Answer the following multiple choice questions regarding the contour plot of a function \( f(x, y) \) below.

(a) If the contour plot models a topographical map of a mountain, describe the experience of a hiker walking above a straight line from \( P \) to \( Q \).
   (i) They walk uphill the whole way.
   (ii) They walk uphill, and then downhill.
   (iii) They walk downhill, and then uphill.
   (iv) They walk downhill the whole way.

(b) Which of the following most accurately describes the values of the first partial derivatives at \( P \)?
   (i) \( f_x(P) > 0, f_y(P) > 0 \);
   (ii) \( f_x(P) > 0, f_y(P) < 0 \);
   (iii) \( f_x(P) < 0, f_y(P) > 0 \);
   (iv) \( f_x(P) < 0, f_y(P) < 0 \).

(c) Which of the following is the best estimate for the direction of the gradient vector \( \nabla f(Q) \)?
   (i) \( (1, 0.5) \);
   (ii) \( (-0.5, -1) \);
   (iii) \( (-1, 2) \);
   (iv) \( (-1, 0.5) \);
Problem 3 (10 points). Consider the function $f(x, y) = 2x^2y^2 - 3xy^3$.

(a) Compute the gradient $\nabla f(1, 1)$.

(b) Write an equation for the tangent plane to the graph $z = f(x, y)$ at the point $(1, 1, -1)$.

(c) Compute the directional derivative $D_v f(1, 1)$ in the direction of the vector $v = \langle 3, -4 \rangle$.

(d) What is the maximum value of the directional derivative at $(1, 1)$?
Problem 4 (10 points). Measure position, \((x, y, z)\), in centimeters for each coordinate. Measure temperature, \(T\), in \(\text{C}^\circ\). Now, suppose that the temperature near a certain point in the atmosphere is modeled by the function

\[ T(x, y, z) = \cos(x + z) \sin(y). \]

(a) If a hummingbird is floating at the origin, fill in the blanks of the vector

\[ \mathbf{v} = \langle \quad , \quad , \quad \rangle \]

so that \(\mathbf{v}\) points in the direction in which the hummingbird should fly in order to increase its surrounding temperature the most (the rate of change will be measured in \(\text{C}^\circ/\text{cm}\)).

(b) Measure time, \(t\), in seconds. Suppose a firefly traverses the path parameterized by the equations

\[ x = \cos(t) \quad y = \sin(t) \quad z = t. \]

After zero seconds, that is \(t = 0\), what rate of change in temperature does the firefly feel in \(\text{C}^\circ/\text{second}\)?