Knowledge Demonstration Opportunity 3: SM221, Calculus III

Name: ______________________

29 March 2018

Read all of the following information before starting:

• You are allowed pencils, pens, your TI-36X calculator, and your wits. That is all. In particular, no computers, notes, books, smartphones, iPads, or pocket-sized hobbits.

• To receive full credit, justify your work clearly and in order. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your “final” answer is correct).

• Use sentences to explain your reasoning. Please keep written answers brief; and simultaneously clear!

• Box or otherwise indicate your final numeric answers.

• Good luck!

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Problem 1 (10 points). Compute the value of the triple integral $\iiint_{E} e^{(x^2+y^2+z^2)^{3/2}} \, dV$ where $E$ is the region in the first octant between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$. 
Problem 2 (10 points). Answer each of the following multiple choice questions.

(a) The picture below shows a conservative vector field $\mathbf{F}$ and a portion of a curve $C$ from the point $P$ to the point $Q$. Assume that $\mathbf{F}$ is defined on all of $\mathbb{R}^2$ and that $C$ is a smooth connected path outside of the depicted region. The value of the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is

(i) positive;
(ii) negative;
(iii) zero;
(iv) can’t tell from the given information

(b) Suppose that $\mathbf{F} = (P, Q)$ is a vector field, defined and differentiable on all of $\mathbb{R}^2$, and that $C$ is a simple closed curve. If $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$, which of the following must be true?

I. $\mathbf{F}$ is conservative
II. $\iint_D (Q_y - P_x) \, dA = 0$, where $D$ is the region enclosed by $C.$

(a) I only
(b) II only
(c) I and II
(d) neither I nor II

(c) Consider the vector field $\mathbf{F}$ depicted below, and the three line segments $C_1$, $C_2$, and $C_3$. Let

$$I_1 = \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \quad I_2 = \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \quad I_3 = \int_{C_3} \mathbf{F} \cdot d\mathbf{r}$$

and choose the correct order for the value of the integrals.

(i) $I_1 < I_2 < I_3$;
(ii) $I_1 < I_3 < I_2$;
(iii) $I_2 < I_1 < I_3$;
(iv) $I_2 < I_3 < I_1$;
(v) $I_3 < I_1 < I_2$;
(vi) $I_3 < I_2 < I_1$;
Problem 3 (10 points). Let $C$ be the unit circle in $\mathbb{R}^2$, oriented counter-clockwise. Compute the following line integrals.

(a) $\int_C (x^2 + y^2) xy^3 \, ds$

(b) $\int_C \langle 3x^2 - 2y, 5x - e^y \rangle \cdot dr$
Problem 4 (10 points). Let $\mathbf{F} = \langle 2xy, x^2 + 2yz, y^2 \rangle$.

(a) Prove that $\mathbf{F}$ is conservative.

(b) Evaluate the line integral $\int_{\Gamma} \mathbf{F} \cdot d\mathbf{r}$ where $\Gamma$ is the curve parameterized by $\mathbf{r}(t) = \langle t^3 + t, t^4 - t^2, te^t \rangle$ for $0 \leq t \leq 1$. 