SM121, Fall 2019, Quiz 2, 04Sep

Name: ________________________________
Period (Circle): 1 or 2 or 3

Directions. Answer the following questions on this piece of paper. Clearly organize your work and indicate your final answers. No calculators or computers are allowed on this quiz.

Let \( f(x) = 5x^2 \).
1. Compute \( f(3) \).
2. Compute the average rate of change for \( f \) on the interval \([3, 4]\).
3. Write a formula for the average rate of change of the function \( f \) on the interval \([3, 3 + \Delta x]\), and simplify it as much as possible (you may assume \( \Delta x \neq 0 \)).
4. Use your formula from Problem 3 to compute the slope of the tangent line to the graph of \( f \) at \( x = 3 \).

1. \[ f(3) = 5 \cdot (3^2) = 5 \cdot 9 = 45 \]

2. \[ \frac{f(4) - f(3)}{4 - 3} = \frac{5 \cdot (4^2) - 45}{1} = \frac{80 - 45}{1} = 35 \]

3. \[ \frac{f(3+\Delta x) - f(3)}{3+\Delta x - 3} = \frac{5 \cdot (3+\Delta x)^2 - 45}{\Delta x} \]

**Aside:** \((3+\Delta x)^2 = 9 + 6 \Delta x + (\Delta x)^2\)

So, average rate of change is:

\[ = \frac{5 \left(9 + 6 \Delta x + (\Delta x)^2\right) - 45}{\Delta x} = \frac{45 + 30 \Delta x + 5 (\Delta x)^2 - 45}{\Delta x} \]

\[ = \frac{30 \Delta x + 5 (\Delta x)^2}{\Delta x} = \frac{\Delta x \left(30 + 5 \Delta x\right)}{\Delta x} = \frac{30 + 5 \Delta x}{\Delta x} \]

4. As \( \Delta x \to 0 \), secant line (aka avg. rate of change) \( \to \) tangent line.

So, slope of tangent line @ \( x = 3 \) is: \( 30 + 5 \cdot 0 = 30 \).
5. Let \( t \) denote time (measured in seconds). Suppose \( g(t) = 5t^2 \) represents the distance (measured in meters) of an electron from the nucleus of a hydrogen atom after time \( t \) has elapsed.

a. Use your answers to Problems 1–4 to determine the distance between the electron and the nucleus after 3 seconds.

b. Use your answers to Problems 1–4 to find the instantaneous velocity of the electron after 3 seconds. Explain your reasoning.

c. What are the units on your answers from Problems 5.a and 5.b?

Notice: \( f(x) \) (from Probs 1-4) is identical to \( g(t) \) after the substitution:

\[ t \leftrightarrow x \]

So

(a) Using the description of \( g \), we see

\[ g(3) = f(3) = \sqrt{45} \text{ m} \]

(b) Using the description of \( g \), the computation of Probs 3 & 4, we see that the instantaneous velocity after 3 sec. have elapsed equals the slope of the tangent line to \( f \) at \( x=3 \).

So

\[ v = \sqrt{30} \text{ m/s} \]

(c) For units... see the answers above, but have:

\[ 5.a \rightarrow \text{meters} \]
\[ 5.b \rightarrow \text{meters per second} \]