We saw in class that to create a parameterized line segment from a point \( a \) to another point \( b \), we can use the vector formula:

\[
r(t) = tb + (1 - t)a.
\]

In three dimensions, if \( a = (x_1, y_1, z_1) \) and \( b = (x_2, y_2, z_2) \), then

\[
r(t) = (tx_2 + (1 - t)x_1, ty_2 + (1 - t)y_1, tz_2 + (1 - t)z_1).
\]

For this choice of \( r \), we have that \( r(0) = a \) and \( r(1) = b \). However, suppose we want more general bounds then \( 0 \leq t \leq 1 \). If we are given \( p < q \) and we want \( r(p) = a \) and \( r(q) = b \) and \( r(t) \) to be a linear vector-valued function, i.e., real bounds \( p \leq t \leq q \).

Give both a vector formula and the three dimensional formula of the correct parameterization. Show your derivation and/or proof that it works.