Bonus problem for Friday, 3/1. 5 bonus points on Test 2. To receive bonus credit you must HAND IN the problems written up by the beginning of class on Friday.

In class, we have been working with the following LP.

\[
\begin{align*}
\text{max} & \quad 10x_1 + 8x_2 + 4x_3 + x_4 \quad \text{(value)} \\
\text{s.t.} & \quad 7x_1 + 5x_2 + 5x_3 + 2x_4 \leq 18 \quad \text{(knapsack capacity)} \\
& \quad x_1, x_2, x_3, x_4 \geq 0
\end{align*}
\]

Recall that we have calculated

\[
x^{(3)} = \begin{pmatrix} 0 \\ 18/5 \\ 0 \\ 0 \end{pmatrix}
\]

and

\[
\hat{d}^{(1)} = \begin{pmatrix} 1 \\ -7/5 \\ 0 \\ 0 \end{pmatrix}, \quad \hat{d}^{(3)} = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{d}^{(4)} = \begin{pmatrix} 0 \\ -2/5 \\ 0 \\ 1 \end{pmatrix}
\]

Let \( y \) be a given vector such that \( y_1, y_2, y_3, y_4 \geq 0 \) and

\[
7y_1 + 5y_2 + 5y_3 + 2y_4 < 18
\]

**To receive bonus credit:**

1. Determine \( w_1, w_2, w_3, \) and \( w_4 \) such that

\[
y = w_2x^{(3)} + w_1\hat{d}^{(1)} + w_3\hat{d}^{(3)} + w_4\hat{d}^{(4)}.
\]

2. Prove your answer is correct.

You must show all work.