Alvin Fine produces three perfumes from raw material. Thirty thousand ounces of raw material is available. Each ounce of raw material can be transformed into 0.4 ounces of perfume 1, 0.3 ounces of perfume 2, and 0.2 ounces of perfume 3, while 0.1 ounces is lost as waste material. Each ounce of perfume 1 can be further processed into 0.6 ounces of perfume 2, 0.3 ounces of perfume 3, and 0.1 ounces of waste material. Alvin Fine has been contracted to produce at least 4000 ounces of perfume 1, 8000 ounces of perfume 2, and 10000 ounces of perfume 3. Because of its environmental initiatives it wishes to minimize waste material. Formulate an LP to minimize waste while producing enough perfume.

Let

\[ R \text{ = ounces of raw material bought} \]
\[ P_1 \text{ = ounces of perfume 1 made and sold} \]
\[ P_{1T} \text{ = ounces of perfume 1 made and further processed} \]
\[ P_2 \text{ = ounces of perfume 2 made and sold} \]
\[ P_3 \text{ = ounces of perfume 3 made and sold} \]
\[ W \text{ = ounces of waste material produced} \]

The formulation is the following.

\[
\begin{align*}
\text{min} & \quad W \\
\text{subject to} & \quad R \leq 30000 & \text{raw material constraint} \\
& \quad 0.4R = P_1 + P_{1T} & \text{perfume 1 balance} \\
& \quad 0.3R + 0.6P_{1T} = P_2 & \text{perfume 2 balance} \\
& \quad 0.2R + 0.3P_{1T} = P_3 & \text{perfume 3 balance} \\
& \quad 0.1R + 0.1P_{1T} = W & \text{waste product balance} \\
& \quad P_1 \geq 4000 & \text{perfume 1 demand constraints} \\
& \quad P_2 \geq 8000 & \text{perfume 2 demand constraints} \\
& \quad P_3 \geq 10000 & \text{perfume 3 demand constraints} \\
R, P_1, P_{1T}, P_2, P_3, W \geq 0 
\end{align*}
\]