Sunco Oil manufactures three types of gasoline (gas 1, gas 2 and gas 3). Each type is produced by blending three types of crude oil (crude 1, crude 2 and crude 3). The sales price per barrel of gasoline and the purchase price per barrel of crude oil are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Sale Price per barrel</th>
<th>Oil</th>
<th>Purchase Price per barrel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas 1</td>
<td>$70</td>
<td>Crude 1</td>
<td>$45</td>
</tr>
<tr>
<td>Gas 2</td>
<td>$60</td>
<td>Crude 2</td>
<td>$25</td>
</tr>
<tr>
<td>Gas 3</td>
<td>$50</td>
<td>Crude 3</td>
<td>$20</td>
</tr>
</tbody>
</table>

Sunco can purchase up to 5000 barrels of each type of crude oil daily. The three types of gasoline differ in their octane rating and sulfur content. The crude oil blended to form gas 1 must have an average octane rating of at least 90 and contain at most 1% sulfur. The crude oil blended to form gas 2 must have an average octane rating of at least 89 and contain at most 2% sulfur. The crude oil blended to form gas 3 must have an octane rating of at least 87 and contain at most 1% sulfur. The octane rating and the sulfur content of the three types of oil are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Octane rating</th>
<th>Sulfur content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crude 1</td>
<td>92</td>
<td>0.5%</td>
</tr>
<tr>
<td>Crude 2</td>
<td>86</td>
<td>2.0%</td>
</tr>
<tr>
<td>Crude 3</td>
<td>90</td>
<td>3.0%</td>
</tr>
</tbody>
</table>

It costs $4 to transform one barrel of oil into one barrel of gasoline, and Sunco’s refinery can produce up to 14,000 barrels of gasoline daily.

Sunco’s customers require the following amounts of each gasoline: gas 1: 3000 barrels per day; gas 2: 2000 barrels per day; gas 3: 1000 barrels per day. The company considers it an obligation to meet these demands. Formulate an LP that will enable Sunco to maximize daily profits (profits = revenues - costs).

**Variables**

\[ x_{11}, x_{12}, x_{13}, \ldots, x_{33} = \text{crude (first index) blended into gas (second index)} \]

\[ c_1, c_2, c_3 = \text{crude (index) bought} \]

\[ g_1, g_2, g_3 = \text{gas (index) produced} \]

\[
\text{max} \quad 70g_1 + 60g_2 + 50g_3 - 45c_1 - 25c_2 - 20c_3 - 4(g_1 + g_2 + g_3)
\]

\[
\text{subject to} \quad g_1 \geq 3000, \quad g_2 \geq 2000, \quad g_3 \geq 1000 \quad \text{(demand)}
\]

\[
0 \leq c_1 \leq 5000, \quad 0 \leq c_2 \leq 5000, \quad 0 \leq c_3 \leq 5000 \quad \text{(purchase)}
\]

\[
92x_{11} + 86x_{12} + 90x_{13} \geq 90g_1 \quad \text{(octane gas 1)}
\]

\[
92x_{12} + 86x_{22} + 90x_{23} \geq 89g_2 \quad \text{(gas 2)}
\]

\[
92x_{13} + 86x_{23} + 90x_{33} \geq 87g_3 \quad \text{(gas 3)}
\]

\[
0.005x_{11} + 0.02x_{12} + 0.03x_{13} \leq 0.01g_1 \quad \text{(sulfur gas 1)}
\]

\[
0.005x_{12} + 0.02x_{22} + 0.03x_{23} \leq 0.02g_2 \quad \text{(sulfur gas 2)}
\]

\[
0.005x_{13} + 0.02x_{23} + 0.03x_{33} \leq 0.01g_3 \quad \text{(sulfur gas 3)}
\]

\[
x_{11}, x_{12}, x_{13}, \ldots, x_{33} \geq 0
\]

\[
c_1 = x_{11} + x_{12} + x_{13}, \quad c_2 = x_{21} + x_{22} + x_{23}, \quad c_3 = x_{31} + x_{32} + x_{33}
\]

\[
g_1 = x_{11} + x_{21} + x_{31}, \quad g_2 = x_{12} + x_{22} + x_{32}, \quad g_3 = x_{13} + x_{23} + x_{33}
\]