Worksheet: Work Scheduling Models, Revisited

Please hand this in on Wed. Feb 18 in class.

1 The postal workers problem, revisited

Recall the postal workers problem:

Example 1. Postal employees in Simplexville work for 5 consecutive days, followed by 2 days off, repeated weekly. Below are the minimum number of employees needed for each day of the week:

<table>
<thead>
<tr>
<th>Day</th>
<th>Employees needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday (1)</td>
<td>7</td>
</tr>
<tr>
<td>Tuesday (2)</td>
<td>8</td>
</tr>
<tr>
<td>Wednesday (3)</td>
<td>7</td>
</tr>
<tr>
<td>Thursday (4)</td>
<td>6</td>
</tr>
<tr>
<td>Friday (5)</td>
<td>6</td>
</tr>
<tr>
<td>Saturday (6)</td>
<td>4</td>
</tr>
<tr>
<td>Sunday (7)</td>
<td>5</td>
</tr>
</tbody>
</table>

We want to determine the minimum total number of employees needed.

Our original model:

Decision variables. Let

\[ x_1 = \text{number of employees who start work on Monday and work through Friday} \]
\[ x_2 = \text{number of employees who start work on Tuesday and work through Saturday} \]
\[ \vdots \]
\[ x_7 = \text{number of employees who start work on Sunday and work through Thursday} \]

Objective function and constraints.

\[
\begin{align*}
\text{min} & \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \\
\text{s.t.} & \quad x_1 + x_4 + x_5 + x_6 + x_7 \geq 7 & \text{(Mon)} \\
& \quad x_1 + x_2 + x_5 + x_6 + x_7 \geq 8 & \text{(Tue)} \\
& \quad x_1 + x_2 + x_3 + x_4 + x_6 + x_7 \geq 7 & \text{(Wed)} \\
& \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_7 \geq 6 & \text{(Thu)} \\
& \quad x_2 + x_3 + x_4 + x_5 + x_6 \geq 4 & \text{(Fri)} \\
& \quad x_3 + x_4 + x_5 + x_6 + x_7 \geq 5 & \text{(Sat)} \\
& \quad x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 & \text{(Sun)} \\
\end{align*}
\]

- Left hand side of (Mon): add up the variables \( x_i \) such that shift \( i \) covers Monday
- We need a way to specify elements of a set that meet certain characteristics
2 Some more set notation

- What if we only want certain elements of a set?
- "::" notation
  \[ j \in S : \text{[condition]} \iff j \in \text{elements of } S \text{ such that } \text{[condition]} \text{ holds} \]
- For example,
  - Define \( N = \{1, 2, 3\}, S_1 = \{a, b\}, S_2 = \{b, c\}, S_3 = \{a, c\}\)
  - Then
    - \( j \in N : j \geq 2 \iff j \in \{2, 3\} \)
    - \( j \in N : a \in S_j \iff j \in \{1, 3\} \).
- Some people use "|" instead "::"

Describe the input parameters of Example 1 using sets and for statements.

Write a linear program for Example 1 using the symbolic input parameters you described above.
## GMPL code

Here is the GMPL model file for this linear program:

```gmpl
# Model for postal employees problem in Lesson 13

## Input parameters ##
set days;                  # days of the week
set shifts;               # shifts
set shift_days{j in shifts}; # days worked for each shift
param required{i in days}; # number of employees needed for each day

## Decision variables and variable bounds ##
var x{j in shifts} >= 0;     # number of employees assigned to each shift

## Objective function ##
# Minimize total number of employees
minimize total_employees: sum{j in shifts} x[j];

## General constraints ##
# Number of employees working on day i >= minimum required on day i
subject to employees_needed{i in days}:
   sum{j in shifts: i in shift_days[j]} x[j] >= required[i];

end;

GMPL data file for this linear program:

```gmpl
# Days of the week
set days := Mon Tue Wed Thu Fri Sat Sun;

# Shifts
set shifts := 1 2 3 4 5 6 7;

# Days worked in each shift
set shift_days[1] := Mon Tue Wed Thu Fri;
set shift_days[2] := Tue Wed Thu Fri Sat;
set shift_days[3] := Wed Thu Fri Sat Sun;
set shift_days[4] := Thu Fri Sat Sun Mon;
set shift_days[5] := Fri Sat Sun Mon Tue;
set shift_days[6] := Sat Sun Mon Tue Wed;
set shift_days[7] := Sun Mon Tue Wed Thu;

# Number of employees needed for each day
param required :=
   Mon 7
   Tue 8
   Wed 7
   Thu 6
   Fri 6
   Sat 4
   Sun 5;
end;
```
SM305 Mid-course Critique
DO NOT PUT YOUR NAME ON THIS SURVEY!!!

I would like your honest feedback regarding SM305 this semester. Your comments will help me make adjustments to improve the course for you and for me.

PLEASE LET ME KNOW WHAT IS OR IS NOT WORKING FOR YOU.

1. My current performance in SM305 is (circle or highlight one):
   - better than I had expected
   - about what I had expected
   - worse than I had expected

2. My performance in this class would improve if the course content were different in the following ways:

3. My performance in this class would improve if the instructor would START or STOP doing the following things:

4. My performance in this class would improve if I would START or STOP doing the following things:

5. Other comments (continue on the back as necessary):