6.1

The gradients are

\[
\nabla(3x_1 + 4x_2 - 6x_3) = \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix} \quad \text{and} \quad \nabla(5x_1 - 2x_2 + 3x_3) = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}.
\]

(a) (i) Since \(\nabla(3x_1 + 4x_2 - 6x_3)^\top d = 3(1) + 4(2) + (-6)(3) = -7 < 0\) and (i) is a maximization problem, \(d\) is not improving.

(ii) Since \(\nabla(5x_1 - 2x_2 + 3x_3)^\top d = 5(1) + (-2)(2) + (3)(3) = 10 > 0\) and (ii) is a minimization problem, \(d\) is not improving.

(b) (i) Since \(\nabla(3x_1 + 4x_2 - 6x_3)^\top d = 3(1) + 4(0) + (-6)(-2) = 15 > 0\) and (i) is a maximization problem, \(d\) is improving.

(ii) Since \(\nabla(5x_1 - 2x_2 + 3x_3)^\top d = 5(1) + (-2)(0) + (3)(-2) = -1 < 0\) and (ii) is a minimization problem, \(d\) is improving.

(c) (i) Since \(\nabla(3x_1 + 4x_2 - 6x_3)^\top d = 3(0) + 4(-1) + (-6)(-1) = 2 > 0\) and (i) is a maximization problem, \(d\) is improving.

(ii) Since \(\nabla(5x_1 - 2x_2 + 3x_3)^\top d = 5(0) + (-2)(-1) + (3)(-1) = -1 < 0\) and (ii) is a minimization problem, \(d\) is improving.