8.2

The decision variable vector is \( x = (x, y, s_1, s_2)^\top \). The constraint matrix is

\[
A = \begin{bmatrix}
2 & -1 & 1 & 0 \\
2 & 1 & 0 & 1
\end{bmatrix}.
\]

The objective function vector is \( c = (3, 2, 0, 0)^\top \). The initial solution and basis are

\[
x^0 = (0, 0, 6, 10)^\top, \quad B^0 = \{s_1, s_2\}.
\]

**Iteration 1:** The simplex directions are

\[
d^x = (1, 0, -2, -2)^\top, \\
d^y = (0, 1, 1, -1)^\top
\]

The reduced costs are

\[
\bar{c}_x = c^\top d^x = 3, \quad \bar{c}_y = c^\top d^y = 2.
\]

The entering variable is \( x \). The step size is \( \lambda_{\text{max}} = 3 \). The leaving variable is \( s_1 \). The new solution and basis are

\[
x^1 = (3, 0, 0, 4)^\top, \quad B^1 = \{x, s_2\}.
\]

**Iteration 2:** The simplex directions are

\[
d^y = (1/2, 1, 0, -2)^\top, \\
d^{s_1} = (-1/2, 0, 1, 1)^\top
\]

The reduced costs are

\[
\bar{c}_y = c^\top d^y = 7/2, \quad \bar{c}_{s_1} = c^\top d^{s_1} = -3/2.
\]

The entering variable is \( y \). The step size is \( \lambda_{\text{max}} = 2 \). The leaving variable is \( s_2 \). The new solution and basis are

\[
x^2 = (4, 2, 0, 0)^\top, \quad B^2 = \{x, y\}.
\]

**Iteration 3:** The simplex directions are

\[
d^{s_1} = (-1/4, 1/2, 1, 0)^\top, \\
d^{s_2} = (-1/4, -1/2, 0, 1)^\top
\]

The reduced costs are

\[
\bar{c}_{s_1} = c^\top d^{s_1} = 1/4, \quad \bar{c}_{s_2} = c^\top d^{s_2} = -7/4.
\]

The entering variable is \( s_1 \). The step size is \( \lambda_{\text{max}} = 16 \). The leaving variable is \( x \). The new solution and basis are

\[
x^3 = (0, 10, 16, 0), \quad B^3 = \{y, s_1\}.
\]

**Iteration 4:** The simplex directions are

\[
d^x = (1, -2, -4, 0)^\top, \\
d^{s_2} = (0, -1, -1, 1)^\top
\]

The reduced costs are

\[
\bar{c}_x = c^\top d^x = -1, \quad \bar{c}_{s_2} = c^\top d^{s_2} = -2.
\]

Therefore, the solution \( x^3 \) is optimal.