inventory

Decision variables. Let

\[ x_1 = \text{amount of product produced in month 1} \]
\[ x_2 = \text{amount of product produced in month 2} \]
\[ \vdots \]
\[ x_{12} = \text{amount of product produced in month 12} \]
\[ I_1 = \text{amount of product held in inventory at the end of month 1} \]
\[ I_2 = \text{amount of product held in inventory at the end of month 2} \]
\[ \vdots \]
\[ I_{12} = \text{amount of product held in inventory at the end of month 12} \]

Objective function and constraints.

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\begin{align*}
\min \quad & (x_1 + \cdots + x_{12}) + \frac{1}{2}(I_1 + \cdots + I_{12}) \quad \text{(total production and inventory cost)} \\
\text{s.t.} \quad & 10 + x_1 = 11 + I_1 \quad \text{(product balance for month 1)} \\
& I_1 + x_2 = 13 + I_2 \quad \text{(product balance for month 2)} \\
& I_2 + x_3 = 23 + I_3 \quad \text{(product balance for month 3)} \\
& I_3 + x_4 = 45 + I_4 \quad \text{(product balance for month 4)} \\
& I_4 + x_5 = 16 + I_5 \quad \text{(product balance for month 5)} \\
& I_5 + x_6 = 32 + I_6 \quad \text{(product balance for month 6)} \\
& I_6 + x_7 = 21 + I_7 \quad \text{(product balance for month 7)} \\
& I_7 + x_8 = 44 + I_8 \quad \text{(product balance for month 8)} \\
& I_8 + x_9 = 17 + I_9 \quad \text{(product balance for month 9)} \\
& I_9 + x_{10} = 32 + I_{10} \quad \text{(product balance for month 10)} \\
& I_{10} + x_{11} = 22 + I_{11} \quad \text{(product balance for month 11)} \\
& I_{11} + x_{12} = 47 + I_{12} \quad \text{(product balance for month 12)} \\
& 0 \leq x_1 \leq 30 \quad \text{(production limits and nonnegativity for month 1)} \\
& \vdots \\
& 0 \leq x_{12} \leq 30 \quad \text{(production limits and nonnegativity for month 12)} \\
& 0 \leq I_1 \leq 40 \quad \text{(inventory limits and nonnegativity for month 1)} \\
& \vdots \\
& 0 \leq I_{12} \leq 40 \quad \text{(inventory limits and nonnegativity for month 12)}
\end{align*}
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