6.9

For this problem, let

$$\mathbf{a}_1 = \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{a}_2 = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}.$$

(a) First, note that

$$\mathbf{a}_1^\top \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} = 6(3) + 2(0) - 1(3) = 15 \quad \text{and} \quad \mathbf{a}_2^\top \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} = 2(3) + 3(0) + 5(3) = 21,$$

so both constraints are active at the solution $(3, 0, 3)$. Then, because

$$\mathbf{a}_1^\top \mathbf{d} = \mathbf{a}_1^\top \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = 6(2) + 2(-1) - 1(0) = 10 \neq 0$$

the direction $\mathbf{d}$ is not feasible.

(b) As calculated previously, both constraints are active at the solution $(3, 0, 3)$. Then, because

$$\mathbf{a}_1^\top \mathbf{d} = \mathbf{a}_1^\top \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = 6(-2) + 2(1) - 1(0) = -10 \neq 0$$

the direction $\mathbf{d}$ is not feasible.

(c) First, note that

$$\mathbf{a}_1^\top \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = 6(2) + 2(2) - 1(1) = 15 \quad \text{and} \quad \mathbf{a}_2^\top \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} = 2(2) + 3(2) + 5(1) = 15 < 21,$$

only the first constraint is active at $(2, 2, 1)$. Then, because

$$\mathbf{a}_1^\top \mathbf{d} = \mathbf{a}_1^\top \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 6(1) + 2(-2) - 1(2) = 0$$

the direction $\mathbf{d}$ is feasible.