

## 7.17

As in 7.16, we assume that the vector solutions have form

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}.$$

- (a) The solution is feasible because it satisfies all constraints. It is basic because the system

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_2 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 6 \end{pmatrix}$$

has a unique solution as the columns are linearly independent. Thus, the solution is a basic feasible solution and an extreme point.

- (b) The solution is not feasible because  $x_5 = -9 < 0$ . However, the equality constraints are satisfied and the system

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{pmatrix} x_2 \\ x_3 \\ x_5 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 6 \end{pmatrix}$$

has a unique solution because the columns are linearly independent. So, the solution is basic but not a basic feasible solution (or an extreme point).

- (c) The solution is feasible and, as in 7.16(c), we must see if there is some choice of basic and nonbasic variables so that the resulting system has a unique solution. The basic variables must include  $x_1$  and  $x_4$ . If we choose  $x_2$  as a basic variable, then the resulting system has linearly independent columns. Thus, the solution is a basic feasible solution, i.e., an extreme point.

- (d) This solution is not feasible because

$$3\left(\frac{1}{2}\right) + 0(1) + 1(1) + 1(0) + 0(2) = \frac{3}{2} + 0 + 1 + 0 + 0 = \frac{5}{2} \neq 5.$$

Thus, it is not basic either. It cannot be a basic feasible solution or an extreme point.

- (e) The solution is feasible because it satisfies all constraints. However, none of the nonnegativity constraints are active (as all components are positive) so it cannot be basic. Therefore, it cannot be a basic feasible solution or an extreme point.