8.2

The decision variable vector is $\mathbf{x} = (x, y, s_1, s_2)^\top$. The constraint matrix is

$$A = \begin{bmatrix} 2 & -1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}.$$

The objective function vector is $\mathbf{c} = (3, 2, 0, 0)^\top$. The initial solution and basis are

$$\mathbf{x}^0 = (0, 0, 6, 10)^\top, \qquad \mathcal{B}^0 = \{s_1, s_2\}.$$

Iteration 1: The simplex directions are

$$\begin{aligned} \mathbf{d}^x &= (1, 0, -2, -2)^\top, \\ \mathbf{d}^y &= (0, 1, 1, -1)^\top \end{aligned}$$

The reduced costs are

$$\bar{c}_x = \mathbf{c}^\top \mathbf{d}^x = 3, \qquad \bar{c}_y = \mathbf{c}^\top \mathbf{d}^y = 2.$$

The entering variable is x. The step size is $\lambda_{\max} = 3$. The leaving variable is s_1. The new solution and basis are

$$\mathbf{x}^1 = (3, 0, 0, 4)^\top, \qquad \mathcal{B}^1 = \{x, s_2\}.$$

Iteration 2: The simplex directions are

$$\begin{aligned} \mathbf{d}^y &= (1/2, 1, 0, -2)^\top, \\ \mathbf{d}^{s_1} &= (-1/2, 0, 1, 1)^\top. \end{aligned}$$

The reduced costs are

$$\bar{c}_y = \mathbf{c}^\top \mathbf{d}^y = 7/2, \qquad \bar{c}_{s_1} = \mathbf{c}^\top \mathbf{d}^{s_1} = -3/2.$$

The entering variable is y. The step size is $\lambda_{\max} = 2$. The leaving variable is s_2. The new solution and basis are

$$\mathbf{x}^2 = (4, 2, 0, 0)^\top, \qquad \mathcal{B}^2 = \{x, y\}.$$

Iteration 3: The simplex directions are

$$\begin{aligned} \mathbf{d}^{s_1} &= (-1/4, 1/2, 1, 0)^\top, \\ \mathbf{d}^{s_2} &= (-1/4, -1/2, 0, 1)^\top. \end{aligned}$$

The reduced costs are

$$\bar{c}_{s_1} = \mathbf{c}^\top \mathbf{d}^{s_1} = 1/4, \qquad \bar{c}_{s_2} = \mathbf{c}^\top \mathbf{d}^{s_2} = -7/4.$$

The entering variable is s_1. The step size is $\lambda_{\max} = 16$. The leaving variable is x. The new solution and basis are

$$\mathbf{x}^3 = (0, 10, 16, 0), \qquad \mathcal{B}^3 = \{y, s_1\}.$$

Iteration 4: The simplex directions are

$$\begin{aligned} \mathbf{d}^x &= (1, -2, -4, 0)^\top, \\ \mathbf{d}^{s_2} &= (0, -1, -1, 1)^\top. \end{aligned}$$

The reduced costs are

$$\bar{c}_x = \mathbf{c}^\top \mathbf{d}^x = -1, \qquad \bar{c}_{s_2} = \mathbf{c}^\top \mathbf{d}^{s_2} = -2.$$

Therefore, the solution $\mathbf{x}^3$ is optimal.