8.8

After putting the given LP in canonical form, the decision variable vector is $\mathbf{x} = (x, y, s_1, s_2)^\top$, the LHS matrix and the RHS vector are

$$A = \begin{bmatrix} -5 & 3 & 1 & 0 \\ 3 & -5 & 0 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 15 \\ 8 \end{bmatrix}$$

and the objective function vector is $\mathbf{c} = (10, -1, 0, 0)^\top$. We can use the following initial BFS and basis:

$$\mathbf{x}^0 = (0, 0, 15, 8)^\top, \qquad \mathcal{B}^0 = \{s_1, s_2\}.$$

Iteration 1: The simplex directions are

$$\mathbf{d}^x = (1, 0, 5, -3)^\top,$$
$$\mathbf{d}^y = (0, 1, -3, 5)^\top,$$

and the reduced costs are

$$\bar{c}_x = \mathbf{c}^\top \mathbf{d}^x = 10, \qquad \bar{c}_y = \mathbf{c}^\top \mathbf{d}^y = -1.$$

So, using Dantzig's rule, the entering variable is x. Using the minimum ratio test, the step size is

$$\lambda_{\max} = \min \left\{ \frac{8}{-(-3)}, \right\} = 8/3,$$

and so the leaving variable is s_2. Therefore, the next BFS and basis are

$$\mathbf{x}^1 = (8/3, 0, 85/3, 0)^\top, \qquad \mathcal{B}^1 = \{x, s_1\}.$$

Iteration 2: The simplex directions are

$$\mathbf{d}^y = (5/3, 1, 16/3, 0)^\top,$$
$$\mathbf{d}^{s_2} = (-1/3, 0, -5/3, 1)^\top,$$

and the reduced costs are

$$\bar{c}_y = \mathbf{c}^\top \mathbf{d}^y = 47/3, \qquad \bar{c}_{s_2} = \mathbf{c}^\top \mathbf{d}^{s_2} = -10/3.$$

Since $\mathbf{d}^y$ is an improving simplex direction with all nonnegative components, the LP is unbounded.