1.72

a) $$A+B=\begin{bmatrix}1+4 & -1+0 & 2-3\\ 0+-1 & 3-2 & 4+3\end{bmatrix}=\begin{bmatrix}5 & -1 & -1\\ -1 & 1 & 7\end{bmatrix}$$

b) A is a 2×3 matrix & C is 3×4.
When adding matrices, they must be the <u>same size</u>.

c) $$3A-4B=\begin{bmatrix}3(1)-4(4) & 3(-1)-4(0) & 3(2)-4(-3)\\ 3(0)-4(-1) & 3(3)-4(-2) & 3(4)-4(3)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -3 & 18\\ 4 & 17 & 0\end{bmatrix}$$

1.73

a) A is (2×3) & B is (2×3).
For multiplication to be valid, the inner dimensions must agree. So

$$\underbrace{(2\times 3)\times(2\times 3)}_{AB}$$ don't match.

b) Note A is (2×3) & C is (3×4)
So the result is

$$AC=(2\times3)\times(3\times4)=(2\times4).$$

$$AC=\begin{bmatrix}2-5-2 & -3+1+0 & 0+4+0 & 1-2+6\\ 0+15+-4 & 0-3+0 & 0-12+0 & 0+6+12\end{bmatrix}$$

$$=\begin{bmatrix}-5 & -2 & 4 & 5\\ 11 & -3 & -12 & 18\end{bmatrix}$$

c) A is (2×3) ; D is (3×1) so result is

$$AD = (2\times\cancel{3})\times(\cancel{3}\times1) = (2\times1)$$

$$AD = \begin{bmatrix} 2+1+6 \\ 0+3+12 \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \end{bmatrix}$$

1.77

a) if $A+B$ is defined then A & B have the same dims. ∴ $B+A$ is defined TRUE

b) False A is (2×3) B is (3×1)
AB is $(2\times\cancel{3})\times(\cancel{3}\times1) = (2\times1)$
but BA is $(3\times1)(2\times3)$
→ not valid

c) FALSE if A is (2×3) then A^T is (3×2)
& $A+A^T$ is not defined.

d) TRUE if A is $(n\times m)$ then A^T is $(m\times n)$
& $AA^T = (n\times\cancel{m})(\cancel{m}\times n) = n\times n$

e) True if AB^T is defined then A is $(m\times n)$
& B^T is $(n\times k)$
& A^T is $(n\times m)$ B is $(k\times n)$ &
$BA^T = (k\times\cancel{n})(\cancel{n}\times m) = k\times m$

2.80: Putting the system into an augmented matrix yields,

a)
$$\left(\begin{array}{cc|c} 2 & 3 & 3 \\ 1 & -2 & 5 \\ 3 & 2 & 7 \end{array}\right) \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 - 3R_2 \end{array} \left(\begin{array}{cc|c} 0 & 7 & -7 \\ 1 & -2 & 5 \\ 0 & 8 & -8 \end{array}\right)$$

$R_1 \leftrightarrow R_2$
$R_2 \to R_2/7$
$R_3 \to R_3/8$

$$\left(\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{array}\right) R_3 \to R_3 - R_2 \left(\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right)$$

This results in $y = -1$ & $x - 2y = 5$

$x - 2(-1) = 5$

$x = 5 - 2 = 3$

So a unique soln of $x = 3, y = -1$.

b) Putting the system into an augmented matrix yields

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 2 & 5 & -8 & 6 & 5 \\ 3 & 4 & -5 & 2 & 4 \end{array}\right) \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & -2 & 4 & -4 & -2 \end{array}\right) R_3 \to R_3 + 2R_2 \left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Infinite solutions satisfying

$x + 2y - 3z + 2t = 2$

& $y - 2z + 2t = 1$

28) c) Putting the system into an augmented matrix yields

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 3 \\ 2 & 4 & 4 & 3 & 9 \\ 3 & 6 & -1 & 8 & 10 \end{array}\right) \begin{array}{l} R_2 - 2R_1 \to R_2 \\ R_3 - 3R_1 \to R_3 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 3 \\ 0 & 0 & 6 & -3 & 3 \\ 0 & 0 & 2 & -1 & 1 \end{array}\right) R_3 \to 3R_3 - R_2$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 3 \\ 0 & 0 & 6 & -3 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Infinite solutions satisfying

$$\begin{cases} x + 2y - z + 3t = 3 \\ 6z - 3t = 3 \end{cases}$$

5

2.82 c) First we put the ~~eqn~~ system into augmented form & row reduce for arbitrary K.

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 1 & -1 & K & -2 \\ 0 & K & 4 & 6 \end{array}\right) R_2 \to R_2 - R_1$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 1 & K & -3 \\ 0 & K & 4 & 6 \end{array}\right) R_3 \to R_3 - KR_2$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 1 & K & -3 \\ 0 & 0 & 4-K^2 & 6+3K \end{array}\right)$$

The last equation yields

$$(4-K^2)z = 6+3K.$$

$\boxed{\text{if } 4-K^2 \neq 0}$ then we can solve for z, $z = 6+3K/(4-K^2)$ & have a unique solution.

If $4-K^2 = 0$ then $K = 2$ or -2.

for $K=2$ the system in echelon form is

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 12 \end{array}\right)$$ since $0 \neq 12$, there is no solution

for $K=-2$ the system in echelon form is

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$ & there are infinitely many solutions

#2. a) The system in augmented matrix form is

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & 5 & 2 & 0 \\ 3 & -1 & -4 & 0 \end{array}\right) \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & -7 & 5 & 0 \end{array}\right) R_3 \to R_3 + 7R_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 61 & 0 \end{array}\right) \to \text{Unique solution } x = y = z = 0$$

Thus basis $= \{\emptyset\}$ dimension is 0.

b) The system in augmented matrix form is

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 2 & 4 & 7 & 1 & 0 \\ 3 & 6 & 10 & 2 & 0 \end{array}\right) \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right) R_3 \to R_3 - R_3$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \to \infty \text{ Solns.}$$

Free vars are y & w

$x + 2y + 3z + w = 0$

$z - w = 0$

$y = 1, w = 0$

$x = -2$
$y = 1$
$z = 0$
$w = 0$

$$\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$y = 0, w = 1$

$z = 1$
$w = 1$
$y = 0$
$x = -4$

$$\begin{pmatrix} -4 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

#2 b) cont

The basis is $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\}$ & dimension is 2.

#3 a) $A\vec{x} = \vec{0}$ in augmented matrix form is

$$\left(\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & -1 & 0 \end{array} \right)$$ This has a unique solution of $x = y = 0$.

$B\vec{x} = \vec{0}$ in augmented matrix form is

$$\left(\begin{array}{cc|c} 2 & 1 & 0 \\ -4 & -2 & 0 \end{array} \right) R_2 \to R_2 + 2R_1 \left(\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right)$$

∞ solutions

b) Note that

$$\left(\begin{array}{cc|c} 2 & 1 & b_1 \\ 0 & -1 & b_2 \end{array} \right)$$ is in echelon form & we can solve

$y = -b_2$ & $2x + y = b_1$

$2x - b_2 = b_1$

$x = \frac{b_1 + b_2}{2}$.

This solution works for any b_1, b_2.

c) we now reduce again

$$\left(\begin{array}{cc|c} 2 & 1 & b_1 \\ -4 & -2 & b_2 \end{array}\right) \quad R_2 \to R_2 + 2R_1 \quad \left(\begin{array}{cc|c} 2 & 1 & b_1 \\ 0 & 0 & b_2+2b_1 \end{array}\right)$$

This either has ∞ solutions (if $b_2 = -2b_1$)
or No solution (if $b_2 \neq -2b_1$).
y is always a free variable.

d) Example if $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ then

$$\left(\begin{array}{cc|c} 2 & 1 & 1 \\ 0 & 0 & 1+2 \end{array}\right) = \left(\begin{array}{cc|c} 2 & 1 & 1 \\ 0 & 0 & 3 \end{array}\right)$$

$0 \neq 3$ thus no solution.

[[Any $\vec{b}$ s.t. $b_2 \neq -2b_1$ would work]]