SM 316 – Spring 2019

Homework 10

Due: Wednesday 24 APRIL 2019

Please read the instructions/suggestions on the course webpage.

Hand in the following problems:

1. From the textbook (Schaum's Outline of Beginning Linear Algebra): 11.57, 11.60 part a), 11.58.

2. Eigenvalues:

   (a) Find the eigenvalues only of the matrix

   $$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

   (b) If a matrix is real and symmetric, that is it has the form

   $$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

   for $a, b, c \in \mathbb{R}$, show that A must have real eigenvalues.

3. Let A be a matrix with eigenvalues $\lambda_1 = 1, \lambda_2 = 2, ..., \lambda_n = n$ and associated eigenvectors $\vec{v}_1, \vec{v}_2, ..., \vec{v}_n$. Let $\vec{b}$ be a vector that can be written as

   $$\vec{b} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n.$$

   Find the constants $d_1, d_2, .., d_n$ such that

   $$A\vec{b} = d_1\vec{v}_1 + d_2\vec{v}_2 + \cdots + d_n\vec{v}_n.$$

4. Let A be a 4×4 matrix with eigenvalues $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = \lambda_4 = -10$ with eigenvectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$, and $\vec{v}_4$. Use a diagonalization of A to find the eigenvalues of, A^n. (Hint: the same diagonalization matrix for A diagonalizes A^n). What happens as $n \to \infty$?