SM 316 – Spring 2019

Homework 2

Due: Friday 25 JAN 2019

PLEASE READ THE INSTRUCTIONS/SUGGESTIONS ON THE COURSE WEBPAGE.

Hand in the following problems:

1. From the text book, 2.52, 2.56, 2.58, 2.73, 2.74, 2.93, 2.94, 2.95, 2.97.

   2.52 Solution: Suppose that in a senior college class of 500 students it is found that 210 smoke, 258 drink alcoholic beverages, 216 eat between meals, 122 smoke and drink alcoholic beverages, 83 eat between meals and drink alcoholic beverages, 97 smoke and eat between meals, and 52 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student:

   a) smokes but does not drink alcoholic beverages;

   For these problems, let S, D, E be the sets of smokers, drinkers and eaters respectively. Then the set of those who smoke but do not drink is $S \cap D'$ which can be written as

$$P(S \cap D') = P(S) - P(S \cap D) = \frac{210 - 122}{500} = \frac{88}{500}.$$

   b) eats between meals and drinks alcoholic beverages but does not smoke;

   We use a similar method as above,

$$P(E \cap D \cap S') = P(E \cap D) - P(E \cap D \cap S) = \frac{83 - 52}{500} = \frac{31}{500}.$$

   c) Neither smokes nor eats between meals.

   This is denoted as

$$P([S \cup E]') = 1 - P(S \cup E) = 1 - (P(S) + P(E) - P(S \cap E)) = \frac{500 - (210 + 216 - 97)}{500} = \frac{171}{500}.$$

   2.56 Solution: An automobile manufacturer is concerned about a possible recall of its best-selling four-door sedan. If there were a recall, there is a probability of 0.25 of a defect in the brake system, 0.18 of a defect in the transmission, 0.17 of a defect in the fuel system, and 0.40 of a defect in some other area.

   a) What is the probability that the defect is the brakes or the fueling system if the probability of defects in both systems simultaneously is 0.15?

   Let $B = \{\text{defect in brakes}\}$, $T = \{\text{defect in transmission}\}$, $F = \{\text{defect in fuel sys.}\}$, $O = \{\text{defect in other}\}$. From inclusion exclusion formula,

$$P(B \cup F) = P(B) + P(F) - P(B \cap F) = 0.25 + 0.17 - 0.15 = 0.27$$

b) What is the probability that there are no defects in either the brakes or the fueling system?

We want to find $P([B \cup C]')$*. Thus*

$$P([B \cup C]') = 1 - P(B \cup C) = 1 - 0.27 = 0.73.$$

2.58 Solution: A pair of fair dice is tossed. Find the probability of getting

a) a total of 8;

Let's put an ordering to the sample space and let one of the die be the first die. Then the sample space is $S = \{(x, y)|x, y \in \mathbb{N}, 1 \le x, y \le 6\}$*. Then the event,* A*, of getting an* 8 *is* $A = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$ *and*

$$P(A) = \frac{5}{36}.$$

b) at most a total of 5.

Using the same sample space as above, we could get a sum of $2, 3, 4$ *or* 5*. This event is* $B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$*, so*

$$P(B) = \frac{10}{36} = \frac{5}{18}.$$

2.73 Solution: If R is the event that a convict committed armed robbery and D is the event that the convict pushed dope, state in words what probabilities are expressed by

a) $P(R|D)$

What is the probability that a convict committed armed robbery given the convict pushed dope.

b) $P(D'|R)$

What is the probability that a convict did not push dope given the convict committed armed robbery.

c) $P(R'|D')$

What is the probability that a convict did not commit armed robbery given the convict has not pushed dope.

2.74 Solution: A class in advanced physics is composed of 10 juniors, 30 seniors, and 10 graduate students. The final grades show that 3 of the juniors, 10 of the seniors, and 5 of the graduate students received an A for the course. If a student is chosen at random from this class and is found to have earned an A, what is the probability that he or she is a senior?

We want to find $P(\textit{student is senior}|A)$*. Using the formula for conditional probability,*

$$P(\textit{student is senior}|A) = \frac{P(\textit{student is senior AND student received an A}}{P(\textit{student received an A})}.$$

We have that 18/50 students received an A thus $P((\text{student received an } A) = 18/50$. *also there were* $10/50$ *students were seniors and got A's. Thus,*

$$P(\textit{student is senior}|A) = \frac{P(\textit{student is senior AND student received an A}}{P(\textit{student received an } A)} = \frac{\frac{10}{50}}{\frac{18}{50}} = \frac{10}{18} = \frac{5}{9}.$$

2.93 Solution: A circuit system is given in Figure 2.11. Assume the components fail independently.

a) What is the probability that the entire system works?

Let A, B, C, D, E *be the events that* A, B, C, D, E *components word, respectively. Then for the circuit to work, either A AND B work OR C AND D AND E work. Using set notation, we need to find* $P(A \cap B \cup (C \cap D \cap E))$. *Using inclusion exclusion,*

$$P(A\cap B\cup(C\cap D\cap E)) = P(A\cap B)+P(C\cap D\cap E)-P(A\cap B\cap C\cap D\cap E) = P(A)P(B)+P(C)P(D)P(E)-P($$

$$= 0.7^2 + 0.8^3 - 0.7^2 0.8^3 = 0.75112.$$

b) Given that the system works, what is the probability that the component A is not working?

If A is not working, then C AND D AND E must be working. Note that we do not care if B is or is not working. Thus, using inclusion exclusion like in part a),

$$P(A'|\textit{working}) = \frac{P(A' \cap (C \cap D \cap E))}{P(\textit{working})}$$

$$= \frac{P(A')P(C)P(D)P(E)}{P(\textit{working})}$$

$$= \frac{0.3 \cdot 0.8^3}{0.75112} = 0.205.$$

2.94 Solution: In the situation of Exercise 2.93, it is known that the system does not work. What is the probability that the component A also does not work?

First note that the probability of not working is

$$P(\textit{working}') = 1 - P(\textit{working}) = 1 - 0.75112 = 0.2488.$$

If A is not working then the component $C \cap D \cap E$ *is not working. Thus,*

$$P(A'|\textit{working}') = \frac{P(A' \cap \cap (C \cap D \cap E)')}{P(\textit{working}')}$$

$$= \frac{P(A')(1 - P(C \cap D \cap E))}{0.2488} = \frac{0.3(1 - 0.8^3)}{0.2488} = 0.588.$$

2.95 Solution: In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, what is the probability that an adult over 40 years of age is diagnosed as having cancer?

Let C be the event that the person has cancer. Let D be the event that the doctor diagnoses cancer. Then from the problem we are given,

$$P(C) = 0.05, \quad P(D|C) = 0.78, \quad P(D|C') = 0.06$$

We want to find the probability $P(D)$. Using the law of total probability, we divide the sample space into those with cancer C and those without C',

$$P(D) = P(D \cap C) + P(D \cap C') = P(C)P(D|C) + P(C')P(D|C') = 0.05 \cdot 0.78 + 0.95 \cdot 0.06 = 0.096.$$

2.97 Solution: Referring to Exercise 2.95, what is the probability that a person diagnosed as having cancer actually has the disease?

This time we want to calculate $P(C|D)$. To do this we use Bayes rule,

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)P(D|C)}{P(C)P(D|C) + P(C')P(D|C')}$$

$$= \frac{0.05 \cdot 0.78}{0.05 \cdot 0.78 + 0.95 \cdot 0.06} = 0.406.$$

2. I have a bag with 3 fair dice. One is 4-side, one is 6-sided, and one is 12-sided. I reach into the bag, pick one die at random and roll it. The outcome of the roll is 4. What is the probability that I pulled out the 6-sided die?

   Solution:*Let D_4, D_6, and D_{12} be the events that I picked the 4,6, or 12 sided dice respectively. The probability we want to find is $P(D_6|4)$. To find this we use Baye's rule by partitioning the space into D_4, D_6, and D_{12}. Thus*

$$P(D_6|4) = \frac{P(D_6 \cap 4)}{P(4)} = \frac{P(D_6)P(4|D_6)}{P(D_4)P(4|D_4) + P(D_6)P(4|D_6) + P(D_{12})P(4|D_{12})}$$

$$= \frac{(1/3)(1/6)}{(1/3)(1/4) + (1/3)(1/6) + (1/3)(1/12)} = \frac{1}{3}$$

3. The Acme Insurance company has two types of customers, careful and reckless. A careful customer has an accident during the year with probability 0.01. A reckless customer has an accident during the year with probability 0.04. 80% of the customers are careful and 20% of the customers are reckless. Suppose a randomly chosen customer has an accident this year. What is the probability that this customer is one of the careful customers.

   Solution:*Let A be the event a customer as an accident. Let R be the event that the chosen customer is reckless and C be the even the customer is careful. We are given*

$$P(A|C) = 0.01, \quad P(A|R) = 0.04, \quad P(C) = 0.8, \quad P(R) = 0.2.$$

We wish to find $P(C|A)$. Using Baye's theorem we have

$$P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{P(C)P(A|C)}{P(C)P(A|C) + P(R)P(A|R)}$$

$$= \frac{0.8 \cdot 0.01}{0.8 \cdot 0.01 + 0.2 \cdot 0.04} = \frac{1}{2}$$