# A Comparison of Two Streamfunction Models of a Meandering Jet


**Abstract**

An equivalent barotropic potential vorticity (PV) model of the Gulf Stream was developed to examine the meandering jet portion of the Stream. The streamfunction equation for the jet can be explicitly defined in terms of x, y, and t. Bower (1991) developed a kinematic model of the Gulf Stream meandering jet. A comparison between the two models show that there is a phase shift and amplitude difference in the jet shapes. The major difference between the two is that the PV model is a dynamic model of the Gulf Stream jet, while Bower's (1991) is strictly a kinematic model. Samelson (1992) modified Bower's (1991) model to observe fluid exchange across a meandering jet. A similar approach is undertaken utilizing our potential vorticity model of the Gulf Stream. The nondimensional streamfunction of the potential vorticity model for motion in the comoving frame is determined and analyzed for comparison to Bower's nondimensional case. Analyses of the systems of differential equations for the two models show that the potential vorticity model is different from the Bower model (1991). The potential vorticity model shows that fluid parcels do not rotate and change orientation slightly as they flow along the meandering jet. However, they do change shape dramatically. The Bower model exhibits a slightly different fluid parcel pattern. The contours of the nondimensional streamfunction for both models are divided up into regions of closed circulation and an eastward propagating jet. Bounding streamlines which connect stagnation points prevent fluid flow across the region boundaries. There is also great differences in the particle paths in the two models.

## 1. Introduction

The Gulf Stream has been the topic of many research efforts for the past several decades. This feature, which flows along the east coast of the United States, is highly dynamic. These studies have ranged from quantitative analyses of the kinematics and dynamics of the Stream to a more qualitative description of the hydrography. Despite the vast amount of work undertaken over the years, the fluid exchange across and within the Gulf Stream is poorly understood. Bower and Rossby (1989) determined that the parcel trajectories in the Gulf Stream exhibit cross-stream motion and are highly correlated with the meandering of the Gulf Stream jet. In the recent past, information about the surface flow structure, fluid exchange and kinematics of the Gulf Stream has been obtained by Bower (1991), Samelson (1992) and Dutkiewicz et al. (1993).

Bower first introduced a simple two-dimensional kinematic model of a meandering jet in 1991. This model was developed to examine the kinematics of fluid exchange within the Gulf Stream region. She compared RAFOS float data with fluid paths in a meandering jet velocity field. This kinematic model for the Gulf Stream region reproduced the Lagrangian observations depicted by the RAFOS float data (Bower, 1991). The main emphasis of her paper was to define a trivial kinematic mechanism for fluid exchange within the meandering jet of the Gulf Stream without including any complicated dynamical processes (Bower, 1991).

Samelson (1992) modified Bower's (1991) model in order to study in detail the exchange of fluid across a meandering jet. The modifications were made in order to determine what deviations in the Gulf Stream meander pattern cause the most efficient exchange of fluid across boundaries. Samelson's model is strictly a kinematic model and has no dynamical constraints.

Mullen (1994) developed a dynamical model of the meandering jet of the Gulf Stream. She undertook this task in order to incorporate some of the dynamics governing the Gulf Stream. Mullen (1994) utilized this nonlinear potential vorticity model to recover the surface flow structure of the meandering jet region of the Gulf Stream through the analyses of satellite imagery and satellite-tracked drifters.

As stated above, the Gulf Stream is a very complicated kinematic, as well as dynamic system. The number of dynamical processes associated with the Gulf Stream is vast. The development of a dynamical model designed to incorporate all these properties would be a very difficult task to undertake. This is true because of the nonlinear relationship among the different dynamical properties governing the flow and meandering of the Gulf Stream. However, the thrust of the paper is the comparison between Bower's kinematic model and Mullen's (1994) potential vorticity model.

The potential vorticity model is discussed in Section 2. The results of the comparison study between Bower's (1991) model results and those of the equivalent barotropic potential vorticity model are presented in the following section. Section 5 contains the summary.

## 2. Dynamic Model

In this paper we will study a model of the stream where the domain in which the fluid flow is defined is divided into three regions. These regions are chosen to signify the following properties of the flow: the upper region, denoted by $\Omega_u$, is where the flow characterizes the evolution of the Sargasso Sea; the lower region, $\Omega_l$, is intended to model the flow in the coastal region; finally, $\Omega_i$, where we expect the dominant features of the flow to occur, models the Gulf Stream region. Let $\Omega = \Omega_l U \Omega_u U \Omega_i$. All subregions consist of semi-infinite domains separated by straight boundaries. Specifically,

$$\Omega = \begin{cases} \Omega_l = \{(x,y)|\, y < -\frac{A}{2}\} \\ \Omega_i = \{(x,y)|\, -\frac{A}{2} < y < \frac{A}{2}\} \\ \Omega_u = \{(x,y)|\, y > \frac{A}{2}\}. \end{cases} \tag{1}$$

We present a nonlinear equivalent barotropic potential vorticity model of the Gulf Stream. Since the bulk of the available data on the stream is of the surface temperature field and since we expect that the streamfunction of the flow will be representative of the surface temperature field in the stream, following Mullen (1994), we introduce a streamfunction formulation of the flow in $\Omega$. Thus, with the same coordinate system as Bower (1991), the governing equation for the dynamic model is the well–known equivalent barotropic potential vorticity equation (Cushman-Roisin, 1993):

$$\frac{\partial q}{\partial t} + J(\Psi, q) = \sigma(x, y, t). \tag{2}$$

Here, $q$ is the potential vorticity and is represented by

$$q = \nabla^2\Psi - \left(\frac{1}{r_d}\right)^2 \Psi + \beta y, \tag{3}$$

where $\Psi$ is the streamfunction and $J$ is the Jacobian of the two functions $\Psi$ and $q$, that is,

$$J(\Psi, q) = \frac{\partial \Psi}{\partial x}\frac{\partial q}{\partial y} - \frac{\partial \Psi}{\partial y}\frac{\partial q}{\partial x},$$

$r_d$ is the Rossby radius of deformation, $f$ is the Coriolis parameter and $\beta = \partial f/\partial y$ represents the constant beta effect. Finally, $\sigma$ is as yet an undetermined potential vorticity source.

Sources are unusual in potential vorticity models. The need for this term arises in our model because the three subregions of $\Omega$ have vastly different physical characteristics. The simplest way we can capture the impact of these charactersitics is by developing solutions of (2) that conserve potential vorticity in each respective subregion, but experience discontinuities across the boundaries. The source provides the jump in potential vorticity as a particle exits one subregion and enters another.

We seek travelling wave solutions to (2) in the form

$$q(x,y,t) = \hat{q}(x-ct,y), \qquad \Psi(x,y,t) = \hat{\Psi}(x-ct,y),$$

whereby the streamfunction is propagating steadily in the $x$–direction. Substituting these forms into (2), we obtain the following relationship

$$J(\hat{\Psi} - cy, \hat{q}) = \sigma(x,y,t). \tag{4}$$

For future reference, let

$$\alpha = \hat{\Psi} - cy. \tag{5}$$

Because the fundamental feature of our physical model is the discontinuity of the potential vorticity in the three regions $\Omega_l$, $\Omega_i$, and $\Omega_u$, we choose the following simple ansatz for $\hat{q}$:

$$\hat{q}(\alpha, y) = Q_l H\left(\frac{-A}{2} - y\right) + Q_u H\left(y - \frac{A}{2}\right) + \\ (-\chi^2\alpha + Q_i)\left(H\left(y + \frac{A}{2}\right) - H\left(y - \frac{A}{2}\right)\right), \tag{6}$$

where $H$ is the standard heaviside function. The quantities $Q_u$, $Q_i$, $Q_u$, and $\chi^2$ are constants and will be determined later from the initial and boundary conditions. It is easy to check that (6) is equivalent to

$$\hat{q} = \begin{cases} Q_l \quad \text{when} \quad y < -\frac{A}{2} \\ -\chi^2\alpha + Q_i \quad \text{when} \quad -\frac{A}{2} < y < \frac{A}{2} \\ Q_u \quad \text{when} \quad y > \frac{A}{2}. \end{cases} \tag{7}$$

Thus, the potential vorticity of this model remains constant in the regions modelling the Sargasso sea and the coastal region but varies linearly in $\alpha$ in the Gulf stream.

Substitution of (6) into (4) gives

$$\sigma(x-ct,y) = -\frac{\partial\alpha}{\partial x}\Big[-Q_l\delta\left(\frac{A}{2} + y\right) + Q_u\delta\left(y - \frac{A}{2}\right) + \\ \left(-\chi^2\alpha + Q_i\right)\big|_{y=-\frac{A}{2}}\delta\left(y + \frac{A}{2}\right) - \left(-\chi^2\alpha + Q_i\right)\big|_{y=\frac{A}{2}}\delta\left(y - \frac{A}{2}\right). \tag{8}$$

Here $\delta$ is the unit impulse function, i.e.,$\frac{dH(z)}{dz} = \delta(z)$.

The last equation empahsizes the point alluded to earlier that a source term in terms of delta functions is needed to conform with the principle of conservation of potential vorticity in the three individual subregions that constitute $\Omega$. It is important to reiterate that potential vorticity is not conserved for particles that enter or leave the region given by $-A/2 \leq y \leq A/2$. Thus, if a particle originally in the Gulf Stream were to wander out into the Sargasso Sea, it would undergo a jump change in its potential vorticity appropriate for that region. Similarly if a particle in the Sargasso Sea were to be entrained by the Gulf Stream, then it also would undergo a jump change in its potential vorticity appropriate for the Stream.

### 3.1 Solution in the Inner Region $\Omega_i$

Solutions to the potential vorticity model in the form

$$\Psi(x, y, t) = Y(y) + \lambda G(y) \sin(k(x - ct)), \tag{9}$$

are sought where $k = 2\pi/L$ is the wavenumber with $L$ as the wavelength. This form was motivated by the kinematic models and has a simple physical interpretation: The solution $\Psi$ consists of a steady background flow, $Y$, upon which $\lambda G \sin$, which describes the meander perturbation, is superimposed. Note that in each region the equations are of the form

$$L_1(G)S + L_2(Y) = 0. \tag{10}$$

From (**??**)

$$\nabla^2 \Psi = -\lambda(k^2 G(y) \sin + G_{yy} \sin + Y'' \tag{11}$$

is obtained, where $k = 2\pi/L$ is the wavenumber with $L$ as the wavelength. Using this result and (6)in (4) gives

$$q = k^2 G(y) \sin + [G_{yy} - \left(\frac{1}{r_d}\right)^2] \sin + Y'' - \left(\frac{1}{r_d}\right)^2 Y + \beta y.$$

For the Inner region the equation to solve is

$$[G_{yy} + (\chi^2 - \left(\frac{1}{r_d}\right)^2 - k^2)G]S + Y_{yy} + [\chi^2 - \left(\frac{1}{r_d}\right)^2]Y - (\chi^2 c - \beta)y - Q_i = 0. \tag{12}$$

From (**??**LGS), it is shown that the Inner region:

$$L_1 = \frac{d^2}{dy^2} + (\chi^2 - \left(\frac{1}{r_d}\right)^2 - k^2), \qquad L_2 = \frac{d^2}{dy^2} - (\chi^2 - \left(\frac{1}{r_d}\right)^2).$$

Since sin is a function of $x$ and $t$, the only way for (12) to be satisfied is for $L_1(G) = L_2(Y) = 0$.

There are discontinuities in the three regions. Therefore, smoothness conditions are necessary. All coefficients are determined by requiring continuity of $\Psi$, $\partial\Psi/\partial x$ and $\partial\Psi/\partial y$ at the boundaries of the three regions. Continuity of $\partial\Psi/\partial x$ and $\partial\Psi/\partial y$ means that the velocities at the boundaries must be continuous. Since the derivatives of both the $Y$ and $G$ portions must be smooth at the upper and lower boundaries of the inner regions, "patches" are required at $y = -A/2$ and $y = A/2$.

The variables for the following equations are $L$, the wavelength of the meander; $k$, the wavenumber; $c$, the phase speed; $r_d$, the radius of deformation; $A$, the amplitude; $M$, the maximum distance of meander; $\beta$, the meridional gradient of the Coriolis parameter; $U$, the velocity ; and $\gamma = \pi/5A$. The subscripts of $u$, $i$ and $l$ represent the different regions of the model domain, the upper, inner and lower regions, respectively.

Concentrating first on the Y portion of the Inner region, the solution to (**??**) is

$$Y_i = A_i \sin(\alpha y) + B_i \cos(\alpha y) + \frac{(\chi^2 c - \beta)y + Q_i}{\alpha^2}. \tag{13}$$

The coefficients $A_i$ and $B_i$ are obtained by focusing on the $u$ velocity equations for the inner region. The velocity relationship for only the $Y$ portion is $U_i = -dY_i/dy$. Here, $U$ represents the $u$ velocity for the $Y$ portion only with the subscript representing the region of interest. Differentiation results in:

$$U_i = -\alpha \left[A_i \cos(\alpha y) - B_i \sin(\alpha y)\right] - \frac{\chi^2 c - \beta}{\alpha^2}. \tag{14}$$

By solving this equation for the conditions at the upper and lower boundaries of the inner region yields

$$U_i\left(\frac{-A}{2}\right) = U = -\alpha A_i \cos\left(\frac{\alpha A}{2}\right) - \alpha B_i \sin\left(\frac{\alpha A}{2}\right) - \frac{\chi^2 c - \beta}{\alpha^2} \tag{15}$$

and

$$U_i\left(\frac{A}{2}\right) = U - \delta = -\alpha A_i \cos\left(\frac{\alpha A}{2}\right) + \alpha B_i \sin\left(\frac{\alpha A}{2}\right) - \frac{\chi^2 c - \beta}{\alpha^2}. \tag{16}$$

Subtracting (12) from (10), the coefficients obtained are

$$A_i = \frac{-U + \delta/2 - (\chi^2 c - \beta)/\alpha^2}{\alpha \cos\left(\frac{\alpha A}{2}\right)}, \qquad B_i = -\frac{\delta}{2\alpha \sin\left(\frac{\alpha A}{2}\right)},$$

where

$$\alpha = \sqrt{\chi^2 - \left(\frac{1}{r_d}\right)^2} > 0.$$

For G Inner, the relationship is defined in terms of cosines

$$G_i = \cos(\gamma y), \tag{17}$$

where $\gamma = \pi/5A$. Substituting $Y$ from (13) and $G$ from (17) into (9) leads to the final form of $\Psi_i$.

### 3.2 Solution in the Lower Region $\Omega_l$

The Lower region has an equation of the form

$$[G_{yy} - (k^2 + \left(\frac{1}{r_d}\right)^2)G]S + Y_{yy} - \left(\frac{1}{r_d}\right)^2 Y - Q_l = 0. \tag{18}$$

Recall that in each region the equations are of the form expressed in (9). Thus, for the Outer regions:

$$L_1 = \frac{d^2}{dy^2} - (k^2 + \left(\frac{1}{r_d}\right)^2), \qquad L_2 = \frac{d^2}{dy^2} - \left(\frac{1}{r_d}\right)^2.$$

The Y portion for $\Omega_u$ and $\Omega_l$ regions are defined in terms of sinh and cosh. Y for the lower portion is presented by

$$\Omega_l = A_l \sinh\left(\frac{y}{r_d}\right) + B_l \cosh\left(\frac{y}{r_d}\right) - Q_l r_d^2. \tag{19}$$

Again the coefficients are obtained by solving the relationship $-dY/dy$. The velocity for the lower region is

$$U_l = U = -\frac{dY}{dy} = -\left(\frac{1}{r_d}\right)\left[A_l \cosh\left(\frac{y}{r_d}\right) + B_l \sinh\left(\frac{y}{r_d}\right)\right]. \tag{20}$$

To meet the required boundary conditions, (20) must match $U_i$ at $y = -A/2$ and it must be equal to zero at $y = -M/2$. This will insure a recirculation pattern in the lower region. By applying the above criteria to (20), the coefficients for the lower region become

$$B_l = \frac{-Ur_d}{\cosh\left(\frac{A}{2r_d}\right)\tanh\left(\frac{M}{2r_d}\right) - \sinh\left(\frac{A}{2r_d}\right)}, \qquad A_l = B_l \tanh\left(\frac{M}{2r_d}\right).$$

The G portion for $\Omega_l$ is shown by the following equations and coefficient definitions where the subscript $lg$ delineates the region of interest. G for the lower portion is defined by

$$\Omega_l = A_{lg} \sinh(\gamma y) + B_{lg} \cosh(\gamma y) \tag{21}$$

where

$$A_{lg} = \frac{\sin\left(\gamma\frac{A}{2}\right) + \cos\left(\gamma\frac{A}{2}\right)\tanh\left(\gamma\frac{A}{2}\right)}{\cosh\left(\gamma\frac{A}{2}\right) - \sinh\left(\gamma\frac{A}{2}\right)\tanh\left(\gamma\frac{A}{2}\right)}, \qquad B_{lg} = \frac{\cos\left(\gamma\frac{A}{2}\right) + A_{lg}\sinh\left(\gamma\frac{A}{2}\right)}{\cosh\left(\gamma\frac{A}{2}\right)}.$$

The coefficients for G of $\Omega_l$ were obtained by requiring that the following relationships be satisfied:

$$G_l\left(\frac{-A}{2}\right) = G_i\left(\frac{-A}{2}\right), \qquad G_l'\left(\frac{-A}{2}\right) = G_i'\left(\frac{-A}{2}\right).$$

Substituting $Y$ from (19) and $G$ from (21) into (9) leads to the final form of $\Psi_l$.

### 3.3 Solution in the Upper Region $\Omega_u$

For $\Omega_u$, the following relationship exists

$$[G_{yy} - (k^2 + \left(\frac{1}{r_d}\right)^2)G]S + Y_{yy} - \left(\frac{1}{r_d}\right)^2)Y - Q_u = 0. \tag{22}$$

The Y portion for $\Omega_u$ is

$$A_u \sinh\left(\frac{y}{r_d}\right) + B_u \cosh\left(\frac{y}{r_d}\right) - Q_u r_d^2. \tag{23}$$

The $U$ velocity equation to solve for the upper region is

$$U_u = U - \delta = \left(\frac{1}{r_d}\right)\left[A_u \cosh\left(\frac{y}{r_d}\right) + B_u \sinh\left(\frac{y}{r_d}\right)\right]. \tag{24}$$

Similar conditions apply for determining the coefficients for the upper region, where (24) equals $U_i$ at $y = A/2$ and zero at $y = M/2$. Thus, the coefficients for this region are

$$B_u = \frac{(U-\delta)r_d}{\cosh\left(\frac{A}{2r_d}\right)\tanh\left(\frac{M}{2r_d}\right) - \sinh\left(\frac{A}{2r_d}\right)}, \qquad A_u = -B_u \tanh\left(\frac{M}{2r_d}D\right).$$

The G portions for the outer upper region is shown by the following equations and coefficients definitions where the subscript $ug$ delineates the region of interest. G for $\Omega_u$ is

$$A_{ug}\sinh(\gamma y) + B_{ug}\cosh(\gamma y), \tag{25}$$

where

$$A_{ug} = \frac{\sin\left(\gamma\frac{A}{2}\right) + \tanh\left(\gamma\frac{A}{2}\right)\cos\left(\gamma\frac{A}{2}\right)}{\sinh\left(\gamma\frac{A}{2}\right)\tanh\left(\gamma\frac{A}{2}\right) - \cosh\left(\gamma\frac{A}{2}\right)}, \qquad B_{ug} = \frac{\cos\left(\gamma\frac{A}{2}\right) - A_{ug}\sinh\left(\gamma\frac{A}{2}\right)}{\cosh\left(\gamma\frac{A}{2}\right)}.$$

The coefficients for the G portion of $\Omega_u$ were obtained by requiring that the following relationships be satisfied:

$$G_u\left(\frac{A}{2}\right) = G_i\left(\frac{A}{2}\right), \qquad G'_u\left(\frac{A}{2}\right) = G'_i\left(\frac{A}{2}\right).$$

Substituting $Y$ from (23)and $G$ from (25) into (9) leads to the final form of $\Psi_i$.

The nondimensional streamfunction in the co-moving frame for the potential vorticity model is

$$\phi(\vartheta, \zeta) = Y_i + GS_i + c2_p\zeta, \tag{26}$$

where

$$Y_i = A1 \sin(\alpha l \zeta) + A2 \cos(\alpha l \zeta] + Y_p,$$

$$Y_p = \frac{(\chi^2 c_p - \beta) l \zeta + Qi}{\alpha^2 \lambda},$$

$$GS_i = \cos(\gamma l \zeta) \sin(k2\vartheta),$$

and and $k2 = kl$, $c2_p = c_p l \lambda^{-1}$, $A1 = A_i \lambda^{-1}$, $A2 = B_i \lambda^{-1}$. $A_i$ and $B_i$ and some of the other parameters were defined previously. The parameter, $l = 40km$, is similar to Samelson's (1992) and Bower's (1991) $\lambda$ term and was used in order to match the scale width of his meandering jet. The nondimensional streamfunction is the focus of this study.

## 4. Comparison of Models

Thus far, the authors have presented the two models and the need for the comparison between the two. In this section, a comparison of the streamfunctions of these two meandering jet models is undertaken.

An experiment was run to compare the behavior of the two different models, Bower's (1991) and Mullen's (1994), utilizing the same initial conditions and parameters. The values used are amplitude = 70 $km$, wavelength = 450 $km$, width = 70 $km$, and phase speed = 14 $km\ dy^{-1}$. The scale factor $\Psi_0$ is calculated by requiring that the maximum current speed be 120 $km\ dy^{-1}$. Within the kinematic model these parameters can be substituted directly into the models and their effects on the streamfunction field can be easily recognized. However, in the potential vorticity model the substitution is not as straightforward. Therefore, a brief discussion concerning these parameters in the dynamic model is necessary before the comparisons can be presented.

Meander wavelength and phase speed are incorporated in the $\sin(k(x - ct))$ component of the solution for the potential vorticity model. The jet width and meander amplitude, however, are not specifically identified in the dynamic model. Rather, they are present implicitly in $A$, $\lambda$ and $\gamma$. Moreover, there is a non-linear coupled relationship between these parameters. This makes it difficult to make a direct comparison between this model and the kinematic model. Thus, a parametric study is undertaken to "tune" the dynamic model by systematically varying $A$, $\lambda$ and $\gamma$ so that a good visual agreement with the kinematic model streamfunction fields is achieved. Overall, the determination of the correct parameter values to substitute into the potential vorticity model is not as straightforward as the direct substitution of parameters into the kinematic model.

A five-day streamfunction field using the same parameters is generated for the two models Mullen (1994). Examination of the results from the two models utilizing the above data shows that there are notable differences. The streamfunction fields are quite different, however the potential vorticity model does meet the requirements of having amplitude and width equal to 70 $km$. Analyses show that there is a sharp difference in the amplitudes of the two models. A phase shift is also evident.

A *Mathematica* program was developed to model the meandering jet of the Gulf Stream. Three fluid parcels are released simultaneously at t = 0 and allowed to follow the meandering jet over a five day time period (Fig. 2). The top panel represents the potential vorticity model run and the bottom shows the results from using Bower's model. Both panels correspond to the fixed frame. The parcels are spaced approximately 20 units distance from each other along the y axis. The time interval between symbols along the trajectory is approximately 0.4 days in both panels. Around each parcel are 20 specific points delineating the boundaries of the parcel. This comparison is made by using the same initial conditions and parameters for both models. The values used are amplitude = 70 $km$, wavelength = 450 $km$, width = 70 $km$, and phase speed = 14 $km\ dy^{-1}$. The scale factor $\Psi_0$ is calculated by requiring that the maximum current speed be 120 $km\ dy^{-1}$. Calculations result in a value of approximately 7257 $km^2\ dy^{-1}$ for $\Psi_0$. Quick analyses of the parcels shows that as the parcels flow along the meandering jet, the fluid parcels are stretched and deformed. However, they do not rotate and only change orientation at certain locations.

Examination of the results from the two models shows that there are notable differences. The streamfunction fields are quite different, however, the PV model does meet the requirement of having an amplitude and width equal to 70 $km$. In the first panel of Fig. 2, which corresponds to the PV model results, three parcels were released at different locations along the y axis and allowed to evolve over time. Starting with the northernmost parcel, the reader can see that there is drastic stretching of the parcel. As this parcel flows from its initial position to the first crest, it begins to stretch in the northern direction allowing it to become slightly elongated. At the crest the particle begins to stretch more and moves at a relatively faster rate as it flows along the eastern flank of the crest. The same type of pattern occurs along the other meander crest and trough combination, where the parcels become more elongated along the flanks and less so at the peaks of the crests and troughs. Also interesting to note is the fact that the parcel remains elongated throughout the entire time period. The northernmost parcel converges towards the stream's center as it flows towards the troughs and diverges slightly at the crests. The orientation of the parcel does change as it flows along the meandering jet. It appears to flow as if the top of the parcel is oriented to the north on the downswing of the crests, while on the upswing the top of the parcel is oriented to the south.

The middle fluid parcel for the PV model is located at the center of the Gulf Stream meandering jet. This parcel does not stretch as much as the northernmost parcel. It also is traveling at greater speeds throughout its course, as evidenced by its ending position 200 $km$ distance in the x direction from the ending point of the upper parcel. The parcel does stretch as it flows along the eastern flanks of the crest. However, on the upswing flank the parcel begins to regain its starting shape. This parcel does not converge and diverge as drastically as the northernmost parcel. It seems to follow the center of the jet quite closely. The orientation of this parcel does not change as much as the northernmost parcel. This fluid packet has its northernmost point always pointing to the top.

Finally, the third fluid parcel in the PV model for the fixed frame follows the southern edge of the meandering jet. This parcel also does not stretch as much as the northernmost fluid packet. Unlike the other two parcels, this fluid parcel stretches the most on the upswing flank of the meanders. As the parcel flows along the first meander it appears to maintain relatively the same shape. However, as it continues to flow over time its shape becomes more distorted and does not return back to its original shape, as was evidenced by the middle parcel. The orientation of this parcel changes only over a long period of time. At the first crest its northern most tip is still oriented to the top. However, as it reaches the second trough the northernmost tip is now oriented toward the bottom. It continues to follow this type of pattern over the remaining time span. The parcel tends to diverge from the center of the stream at troughs and converges near the crests. This parcel travels at about the same speed as the upper parcel. This can be seen by the location of its termination point for the time span.

The bottom panel of Fig. 2 displays the fluid parcel path for the Bower model (1991) with the same parameters as the PV model. These three parcels follow a similar pattern of stretching and orientation changes. However, there are some other aspects of these parcels which are not seen in the PV model results. The stretching of this parcels do not occur until well after the first crest. Before this, they maintain their original shape and only change their orientation slightly. Observing the northernmost parcel, one can see that the parcel follows a similar pattern as the northernmost fluid parcel in the PV model. It stretches dramatically along the first flank, then decreases a little in the stretching on the upswing side. After that, the parcel remains stretched. The parcel tends to converge at the troughs and diverge at the crests. Again, the orientation is similar to the PV model. This parcel has its northernmost tip oriented to the south when it reaches the meander troughs and continues to flow up the western flanks of the meanders in that orientation.

The fluid parcel in the center of the jet maintains its original shape much better than the other two parcels. Despite the fact that it is in the center of the jet, where the speeds should be greatest, the parcel does not travel much farther than its counterparts for the same time period. The center fluid parcel appears to neither converge nor diverge along its path. Its orientation also remains in its original position, with only very slight variations at the crests and troughs. This parcel does not stretch as much as the parcels located to its north and south.

The southernmost parcel does undergo shape and orientation changes in the Bower model (1991). It becomes elongated on the upswing of the meanders. At the troughs it is evident that the orientation of the northern tip of the parcel has shifted with it now pointing to the south. The parcel converges along the flanks and diverges at the troughs. The parcel speeds appear to be similar to the other parcels. All three parcels have a termination point right around 1400 $km$.

There are a couple of major differences between the Bower (1992) and PV model parcel paths. The distance traveled by the all three parcels in the PV model are much greater than in the Bower model. The PV parcels obtain a distance of approximately 1700 $km$, while the Bower model parcels reach only 1400 $km$ for the same time period. Obviously, the orientation and elongation of the parcels in each model are different. The parcels in the PV model tend to be more elongated and change their orientation more often along its course. Both models display convergence and divergence patterns for their three parcels.

Figure 3 displays the nondimensional particle trajectories for both the potential vorticity model and Bower's model in the co-moving frame. Despite the fact that the same set of constraints were used in each model, one can see the drastic difference between the kinematic and dynamic models. The parameters used are the same as in Fig. 2, however they have been nondimensionalized. The first panel represents the non-d trajectories for the potential vorticity model. Forty initial points were started at time zero on the y axis and were run for a period of 15 units of time. The results from the potential vorticity model are very intriguing in that the particles flow in a negative as well as a positive direction, where the x direction is positive eastward and in the y direction is positive northward. The trajectories flowing to the east do follow the meandering jet pattern. It is interesting to note that there is a recirculation area at the base of the first crest. The meandering jet pattern narrows at the flanks and broadens again at the crests and trough regions. Also evident is the change in spacing between the particle paths as one progresses farther north or south. It increases in the northerly and southerly directions. The most interesting part of the potential vorticity model runs is the fact that the particle trajectories also flow to the west. However, this occurs only at the regions that appear to be outside the confines of the meandering jet portion. Again, a recirculation region has developed to the right of the first crest. In this region, one can see how the particles began flowing eastward and then changed course by flowing westward into a recirculation core. At the outer boundaries of the model the particle trajectories follow a slight meandering pattern to the left.

The bottom panel of Fig. 3 displays the results from Bower's model. The non-d axes are the same as the potential vorticity model, as well as the time and initial conditions. One can see that the meandering pattern is drastically different from the dynamic model. The majority of the flow is in the positive x direction, with some flow going to the west. Bower's model follows the same meandering pattern as the potential vorticity model. However, its crests and troughs are not as peaked. They are more rounded. The particle trajectories are also evenly spaced, only widening a little at the troughs. The meandering jet pattern in the Bower model is different from the PV model in the fact that it travels through two meander crests over the same time period. The PV model only approaches the flank of the second crest, while the Bower model is already on the downswing leg for the second crest. Evident in the Bower model are three closed circulation systems near the crests and troughs. It also has bounding streamlines above and below the crests and troughs, respectively. Clearly, there is a amplitude and meander phase shift between the two model results. The differences between the two model's particle trajectories show that the potential vorticity model has succeeded in incorporating more of the gulf stream dynamics. This will aid in the understanding of particle flow within and across the meandering jet. It was important to plot the two model results on different scales in order to see that there is a drastic difference between the two meandering jets for the same parameters and initial conditions. The different scales allow one to see that the PV model particles do flow in the westerly direction, as well as in the positive x direction.

The final figure, Fig. 4, shows the contours of the non-d streamfunction, $\phi$, in the co-moving frame. The contours in both panels correspond to the streamlines within the moving frame. They also describe the flow because the system is time independent. The contours for the potential vorticity streamfunction are shown in the upper panel. Clearly, the eastward meandering jet is illustrated with its increased spacing as one approaches the peak of the crests and troughs. The potential vorticity model can be divided into distinct regions. The regions represent the meandering jet and two closed circulation systems near the crest and trough. Another interesting feature to note in the dynamic model is the presence of bounding streamlines which connect stagnation points above the crests and below the troughs. These streamlines prevent flow from going from one region to the other.

The lower panel illustrates the streamfunction contours for the kinematic model. Bower's flow model is divided into three regions: a centralized eastward flowing meandering jet, exterior retrograde relative motion, and a closed circulation system at the crests and troughs (Samelson, 1992). The meandering jet region is evenly spaced. It is interesting to note that over the same time span the meandering jet of Bower displays two meander crests. This streamfunction plot shows three closed circulation regions, as well as the meandering jet pattern. Two bounding streamlines separate the regimes preventing any fluid exchange between the closed circulation regions and the meandering jet. As stated by Samelson (1992), these bounding streamlines connect stagnation points which are found above the meander crests and below the troughs.

The two models display fairly similar streamfunction plots. Each shows a eastward meandering jet with closed circulation regions near the crests and troughs. However, a drastic dissimilarity is the fact that Bower's model exhibits two meander crests for the same time and space scales, while the PV model only displays one. This shows that despite the fact that the dynamic model was "tuned" to visually agree with the kinematic model, the resulting meander wavelengths are different by a factor of two. They both are also bounded by streamlines at their northern and southern regions. As stated earlier, these bounding streamlines connect stagnation points and prevent the flow of fluid across the boundaries and the exchange of fluid between the different regions.

### 5. Summary

Contours and particle trajectories of the nondimensional streamfunction for two separate models have been investigated. One of the models is a nonlinear equivalent barotropic potential vorticity model of a meandering jet, while the other is strictly a kinematic model of the same jet. The results indicate that there are some strong similarities and differences between the two models. The non-d particle trajectory plots for each model show that there are distinct differences. The potential vorticity model displays a plot which illustrates the dynamics of the meandering jet stream. The fluid parcels flow in the westerly, as well as the easterly direction, with two major recirculation regions near the origin. However, in the kinematic model, where the dynamics of the system is discarded, the particle trajectories are mainly flowing to the east in a meandering jet fashion.

Parcel trajectories for the models are similar. Two of the three parcels within the models converge and diverge. However, the center parcel appears to flow strictly along the center streamline. Parcel orientation and shape change dramatically along the meander flanks in both models. Again, the center parcel does not deform as much in its shape. A major difference in these models is the fact that the PV model parcel trajectories reach a length of approximately 1700 $km$ and the Bower model trajectories only approach 1400 $km$ spanning over the same time period.

One of the relative dissimilarities between the two models is the figure displaying the contours of the nondimensional streamfunction. Both models, kinematic and dynamic, depict a eastward flowing meandering jet along with closed circulation systems near the crests and troughs of the jet. However, Bower's model depicts two meandering crests and troughs for the same time period as Mullen's, which only produces one crest/trough combination. This is despite the fact that certain variables of the dynamic model were systematically varied to obtain a good visual match with the kinematic model. Also, important to note is the fact that there are bounding streamlines connecting stagnation points located above and below the meander crests and troughs. Most probably this dynamic model would be able to provide great insight into a better understanding of the cross-frontal exchange than the previous kinematic models. Clearly, the PV model gives a better representation of the Gulf Stream and its dynamics than the Bower (1991) model.

The application of dynamical systems theory (DST) to study the fluid exchange across and within the meandering jet of the Gulf Stream will be a future study to be pursued by these authors. Geometric dynamical systems theory is an important tool in studying Lagrangian trajectories. A major premise of the dynamical systems theory is that some geometric structures can be isolated which establish the structure of the entire flow (Jones et al., 1995). Particularly, there are certain trajectories that delineate qualitatively different regions of trajectory motion. Jones et al. (1995) show that the organization of the fluid flow is partially governed by the stagnation points of the flow and stable and unstable manifolds. By using different dynamical systems techniques, one will be able to determine whether the system of equations is chaotic or not.

Perturbations to the meandering jet streamfunction were undertaken by Samelson (1992) in his modification of Bower's model. He did this in order to answer the question of what deviations of the Gulf Stream would cause fluid exchange to occur between the model regions. Melnikov's (1963) method is such a perturbation technique which was developed to research the nonlinear stability of dynamical systems and was the method utilized by Samelson. This analytical method is used to determine whether intersections of stable and unstable manifolds exist in a system (Ottino, 1989). This technique also is used to determine the splitting of separatrices under small perturbations of nonlinearities, $\epsilon$. The question to ask is what happens to the fluid parcel if the flow varies only slightly from the meandering jet pattern. The authors mention this because a continued study of this potential vorticity model would be to apply a perturbations technique to the meandering jet of this model in order to determine the fluid exchange across the jet and see whether their conclusions agree with Samelson's (1992) model results.

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## FIGURE CAPTIONS

FIG. 1. Particle trajectories within the meandering jet in the fixed frame for the potential vorticity model (top panel) and the kinematic model (bottom panel). Three particles are released simultaneously from the y axis. The time interval between symbols for each trajectory is approximately 0.4 days.

FIG 2. Particle trajectories for both Mullen's (1994) (top panel) and Bower's (1991) (bottom panel) models in the nondimensional co-moving frame. Both panels depict evolution over the same time period of 15 units of time.

FIG 3. Contours of the nondimensional streamfunction in the moving frame. The top panel represents the streamfunction for Mullen (1994), while the bottom depicts results from Bower's (1991) model.

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