Example 3. You are a portfolio manager in charge of a bank portfolio with $10 million to invest. There are 5 different securities available:

<table>
<thead>
<tr>
<th>Bond name</th>
<th>Bond type</th>
<th>Years to maturity</th>
<th>Rate of return at maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Municipal</td>
<td>9</td>
<td>4.3%</td>
</tr>
<tr>
<td>2</td>
<td>Agency</td>
<td>15</td>
<td>2.7</td>
</tr>
<tr>
<td>3</td>
<td>Government</td>
<td>4</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>Government</td>
<td>3</td>
<td>2.2</td>
</tr>
</tbody>
</table>

The bank has some policies that limit how you can construct your portfolio:

1. Municipal and agency bonds must total at least $4 million
2. The average years to maturity of the portfolio must not exceed 5 years
3. Bonds cannot be “shorted” (cannot buy negative amounts of bonds)

Write a linear program that determines a portfolio of the above securities that maximizes earnings.

DVs: \( x_1 = \text{amt. invested in bond 1, in millions} \)

Define \( x_2, x_3, x_4, \) similarly

\[
\begin{align*}
\text{max} \quad & 0.043x_1 + 0.027x_2 + 0.025x_3 + 0.022x_4 \quad \text{(total earnings)} \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \leq 10 \quad \text{(budget)} \\
& x_1 + x_2 \geq 4 \quad \text{(M+A requirement)} \\
& 9x_1 + 15x_2 + 4x_3 + 3x_4 \geq 5(x_1 + x_2 + x_3 + x_4) \quad \text{(avg YTM req.)} \\
& x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0, \ x_4 \geq 0 \quad \text{(no shorting)}
\end{align*}
\]